A new procedure of analyzing four-body breakup reaction of the Borromean halo nucleus ^{6}He

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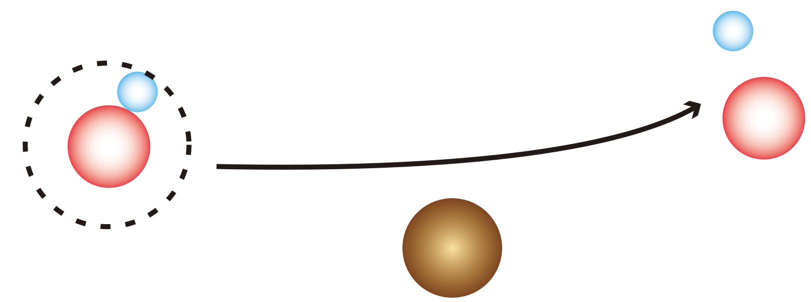
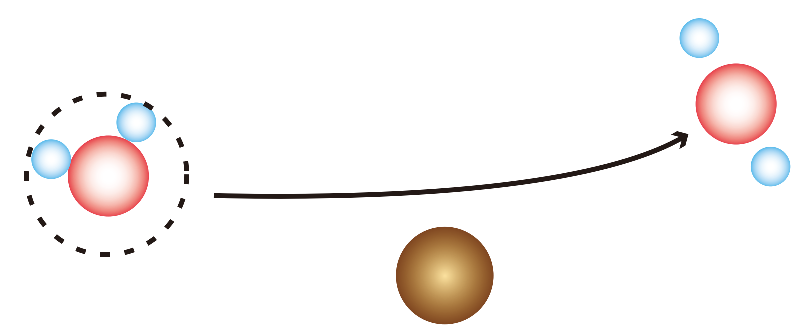
Introduction

- Continuum Discretized Coupled Channels method
  - three-body CDCC
    - ✓ elastic scattering (nuclear+Coulomb)
    - ✓ breakup reaction (nuclear+Coulomb)

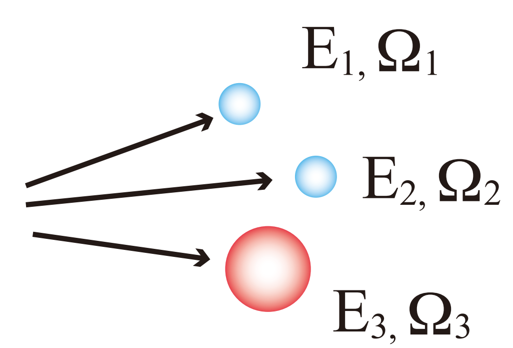


  - four-body CDCC
    - ✓ elastic scattering (nuclear+Coulomb)
    - \- breakup reaction (nuclear+Coulomb)



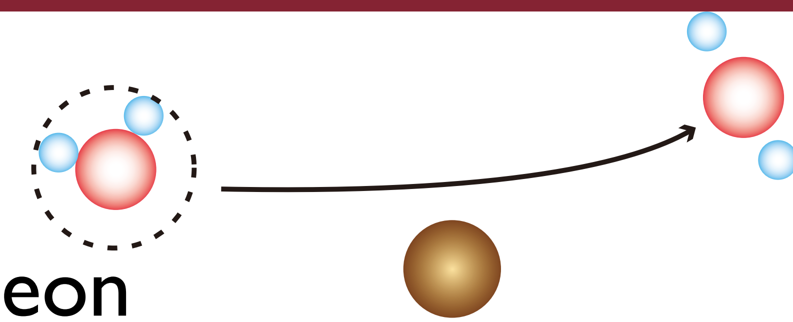
quinted cross section

$$\frac{d^5\sigma}{dE_1 dE_2 d\Omega_1 d\Omega_2 d\Omega_3}$$



Introduction

Four-body breakup reaction of ^{6}He



- GSI: ^{6}He scattering on C, Pb @ 240 MeV/nucleon

  T. Aumann et al., Phys. Rev. C59, 1252 (1999).

- MSU: ^{6}He scatteirng on C, Al, Cu, Su, Pb, U @ 23.9 MeV/nucleon

  J. Wang et al., Phys. Rev. C61, 034301 (2001).

- 3-body correlation of 3 fragments of ^{6}He+Pb @ 240A MeV

  exp: L.V. Chulkov et al., Nucl. Phys. A759, 23 (2005).

  cal: S.N. Ershov et al., Phys. Rev. C74, 014603 (2006).

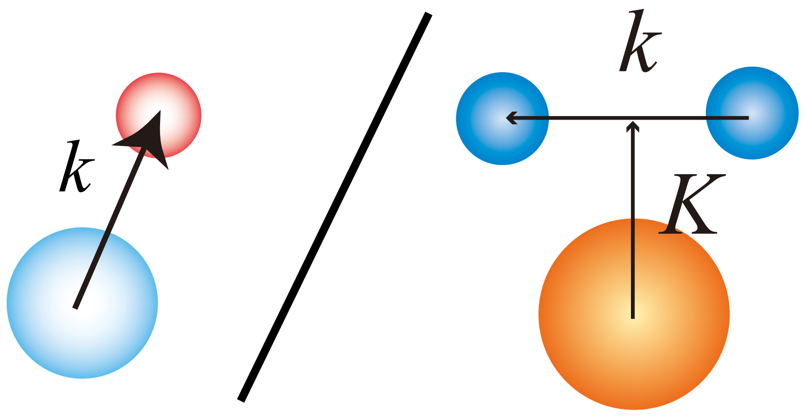
Hyperspherical harmonics method could not reproduce the correlation data.

Analysis by four-body CDCC is necessary.

Purpose

In order to calculate the four-body breakup cross sections, we need breakup S-matrix element *S*(*k*) as a continuous function of momentum *k* between constituent particles.

breakup cross section



$$\sigma^{bu}(k) \propto |S(k)|^2$$ (3-body CDCC)

$$\sigma^{bu}(k, K) \propto |S(k, K)|^2$$ (4-body CDCC)

We have to make the breakup S-matrix element obtained by CDCC calculation a continuous function as a function of *k*.

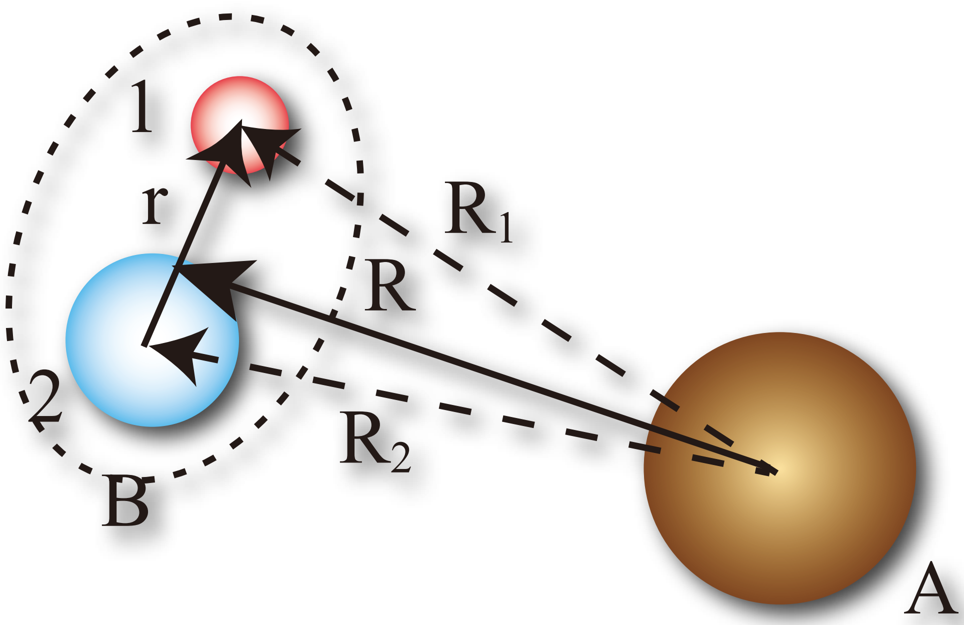
breakup S-matrix element

$$\hat{S}_i \rightarrow S(k)$$ (3-body CDCC)

$$\hat{S}_i \rightarrow S(k, K)$$ (4-body CDCC)

three-body CDCC

M. Kamimura et al., Porg. Theor. Phys. Suppl. 89,1 (1986).
N. Austern et al., Phys. Rep. 154, 125 (1987).



Schrödinger equation

$$[H - E]\Psi = 0$$

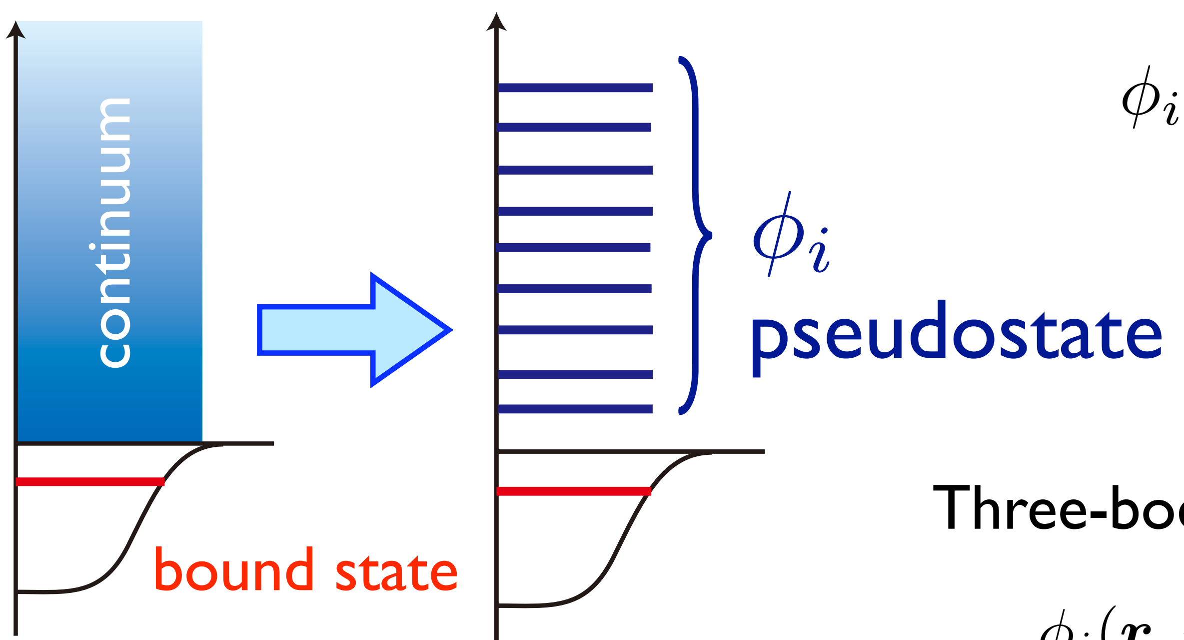
Hamiltonian

$$H = T_R + U_1(R_1) + U_2(R_2) + h_B$$
$$h_B = T_r + V_{12}(r)$$

Discretization of internal state

← Gaussian Expansion Method

E. Hiyama, Y. Kino, M. Kamimura, Prog. Part. Nucl. Phys. 51, 233 (2003).



$$\phi_i(\boldsymbol{r}) = \sum_n C_n^{(i)} r^\ell e^{-\nu_n r^2} Y_\ell(\hat{\boldsymbol{r}})$$

$$\langle \phi_i | h_B | \phi_j \rangle = \epsilon_i \delta_{ij}$$

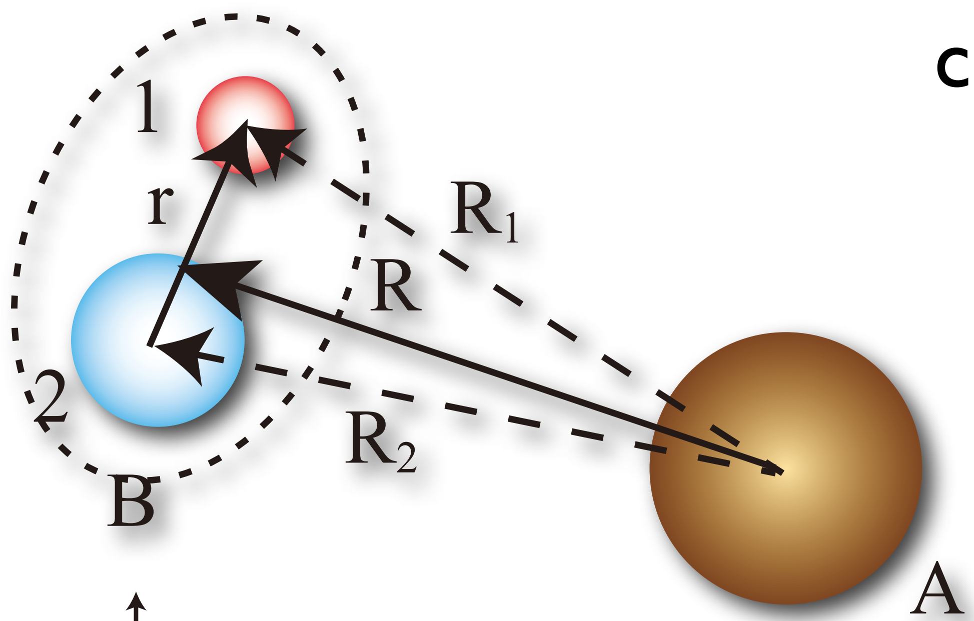
$$\langle \phi_i | \phi_j \rangle = \delta_{ij}$$

Three-body projectile (four-body CDCC)

$$\phi_i(\boldsymbol{r}, \boldsymbol{y}) = \sum_n C_n^{(i)} r^\ell e^{-\nu_n r^2} R^L e^{-\lambda_n y^2} [Y_\ell(\hat{\boldsymbol{r}}) \otimes Y_L(\hat{\boldsymbol{y}})]_{Im}$$

Continuum Discretized Coupled Channels method

- three-body CDCC



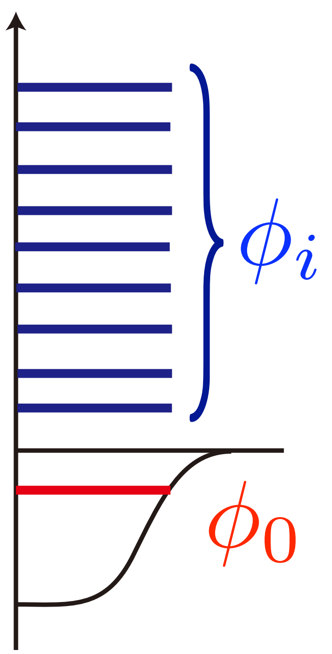
Total wave function is expanded by complete set of internal wave function

$$\Psi = \phi_0(\boldsymbol{r})\chi_0(\boldsymbol{R}) + \sum_i \phi_i(\boldsymbol{r})\chi_i(\boldsymbol{R})$$

Coupled channel equation

$$[T_R + F_{ii}(R) + (\varepsilon_i - E)]\chi_i(R) = -\sum_{i\neq j} F_{ij}(R)\chi_j(R)$$

$$F_{ij}(R) = \langle \phi_i | U_1 + U_2 | \phi_j \rangle$$



boundary condition

$$\chi_i(R) \rightarrow U^{(-)}(R) - \sqrt{\frac{P_0}{P_i}}\hat{S}_i U^{(+)}(R)$$

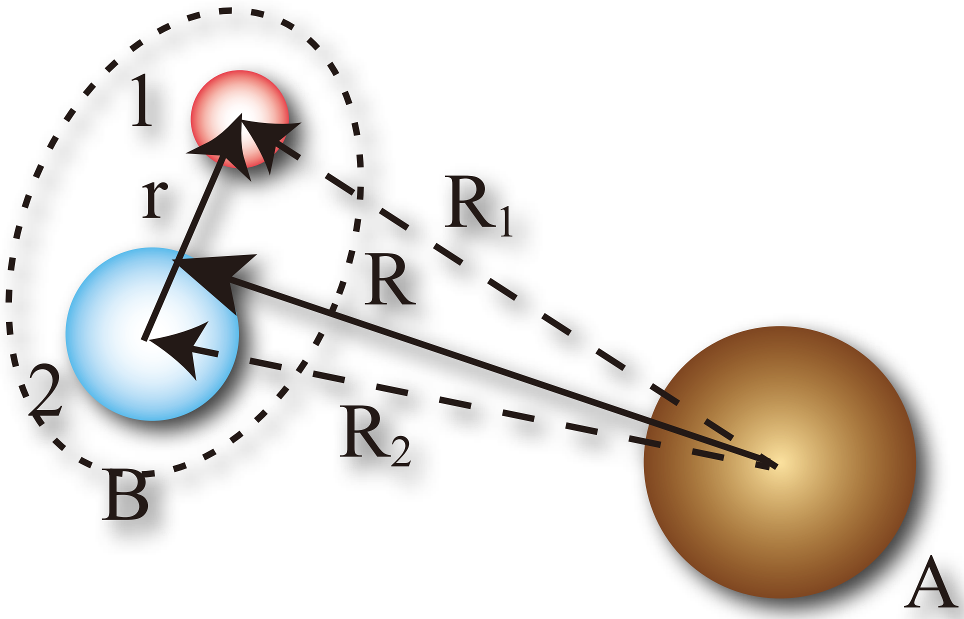
In order to calculate breakup cross section $\sigma^{bu}(k) \propto |S(k)|^2$

we have to make S-matrix as a smooth function. $\hat{S}_i \longrightarrow S(k)$

Smoothing S-matrix

- two-body projectile

T. Matsumoto et al., PRC68, 064607 (2003).



S-matrix obtained by CDCC

$$\hat{S}_i \sim \langle \phi_i(r) F(R) | V | \Psi^{\mathrm{CDCC}}(r, R) \rangle$$

Exact solution of S-matrix

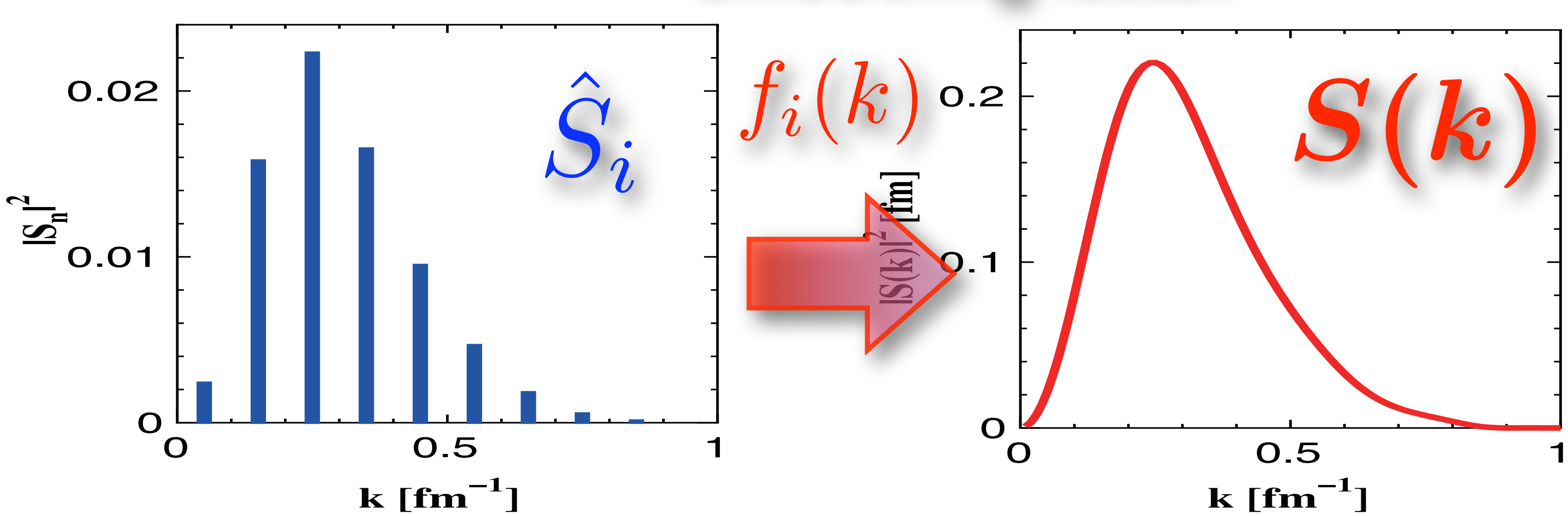
$$S(k) \sim \langle \psi(k, r) F(R) | V | \Psi^{\mathrm{CDCC}}(r, R) \rangle$$

$$S(k) \approx \sum_i \langle \psi(k, r) | \phi_i(r) \rangle \langle \phi_i(r) F(R) | V | \Psi^{\mathrm{CDCC}} \rangle$$

complete set

$$= \sum_i f_i(k) \hat{S}_i$$

smoothing factor



Problem of smoothing

- Three-body CDCC (two-body projectile)

$$f_i(k) = \langle \psi(k) | \phi_i \rangle$$

We can get the continuum wave function of two-body system.
We can obtain smoothing factor easily.

- Four body CDCC (three-body projectile)

$$\mathcal{F}_i(k, K) = \langle \psi(k, K) | \phi_i \rangle$$

It is difficult to obtain the continuum wave function of three-body system. We can not obtain smoothing factor and breakup S-matrix as a smooth function of energy.

Another method is necessary to obtain smoothing factor.

Model Space Approximation

Lippmann-Schwinger equation

Definition of projection operator

$$P = \sum_i |\phi_i\rangle\langle\phi_i| \qquad \langle\phi_i|h_B|\phi_j\rangle = \epsilon_i\delta_{ij}$$

Using projection operator P, we write continuum wave function.

$$P|\psi\rangle = P|\psi_0\rangle + P\frac{1}{\varepsilon - h_0 + i\epsilon}PVP|\psi\rangle \qquad [h_0 - \varepsilon]|\psi_0\rangle = 0$$

Multiplying a pseudostate by the left side

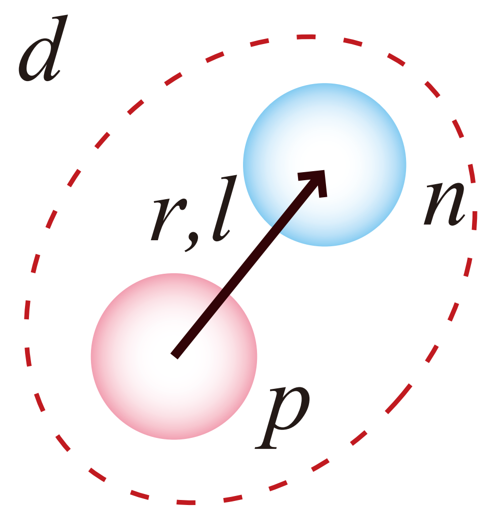
$$\langle\phi_i|\psi\rangle = \langle\phi_i|\psi_0\rangle + \sum_{jk}\langle\phi_i|\frac{1}{\varepsilon - h_0 + i\epsilon}|\phi_j\rangle\langle\phi_j|V|\phi_k\rangle\langle\phi_k|\psi\rangle$$

$$f_i(k) = \langle\phi_i|\psi\rangle^*$$

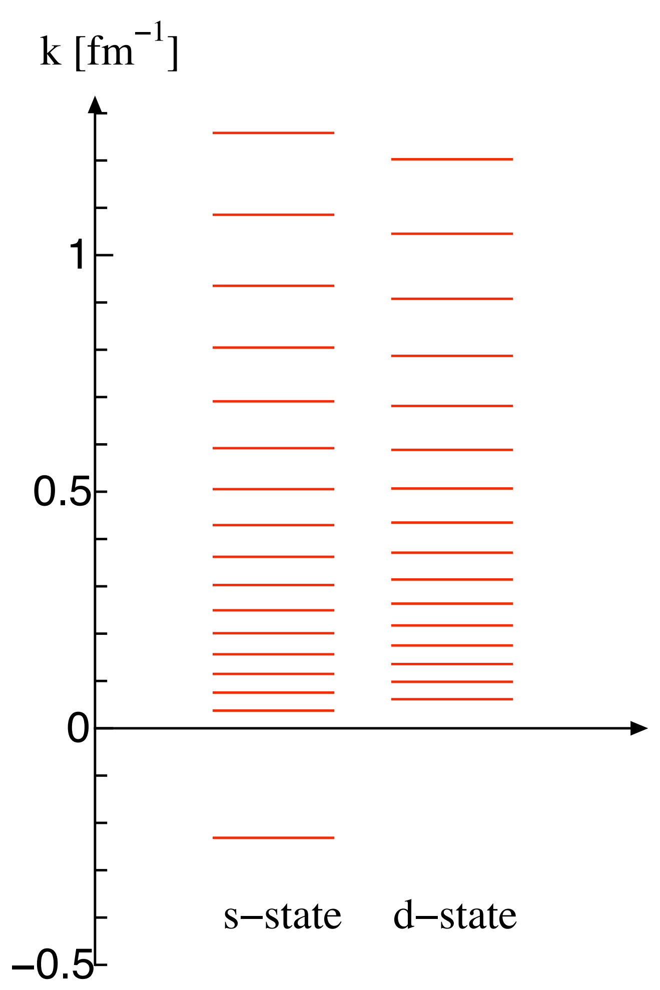
$$f_i(k)^* = \langle\phi_i|\psi_0\rangle + \sum_{jk}\langle\phi_i|\frac{1}{\varepsilon - h_0 + i\epsilon}|\phi_j\rangle\langle\phi_j|V|\phi_k\rangle f_k(k)^*$$

Solving this simultaneous equation, we can obtain smoothing factor.

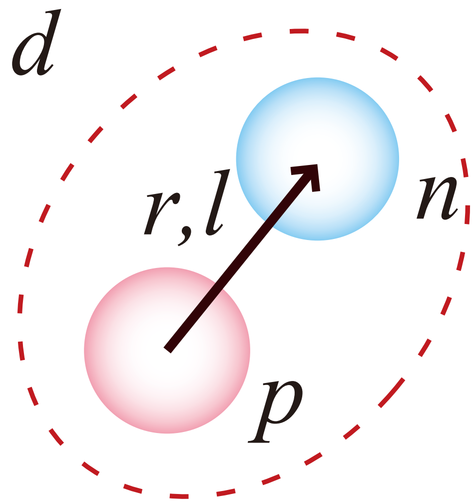
In order to check the validity of the new method, we compared the result of old method with exact continuum wave functions with that of new one based on Lippmann-Schwinger equation.



- smoothing factor
- breakup S-matrix

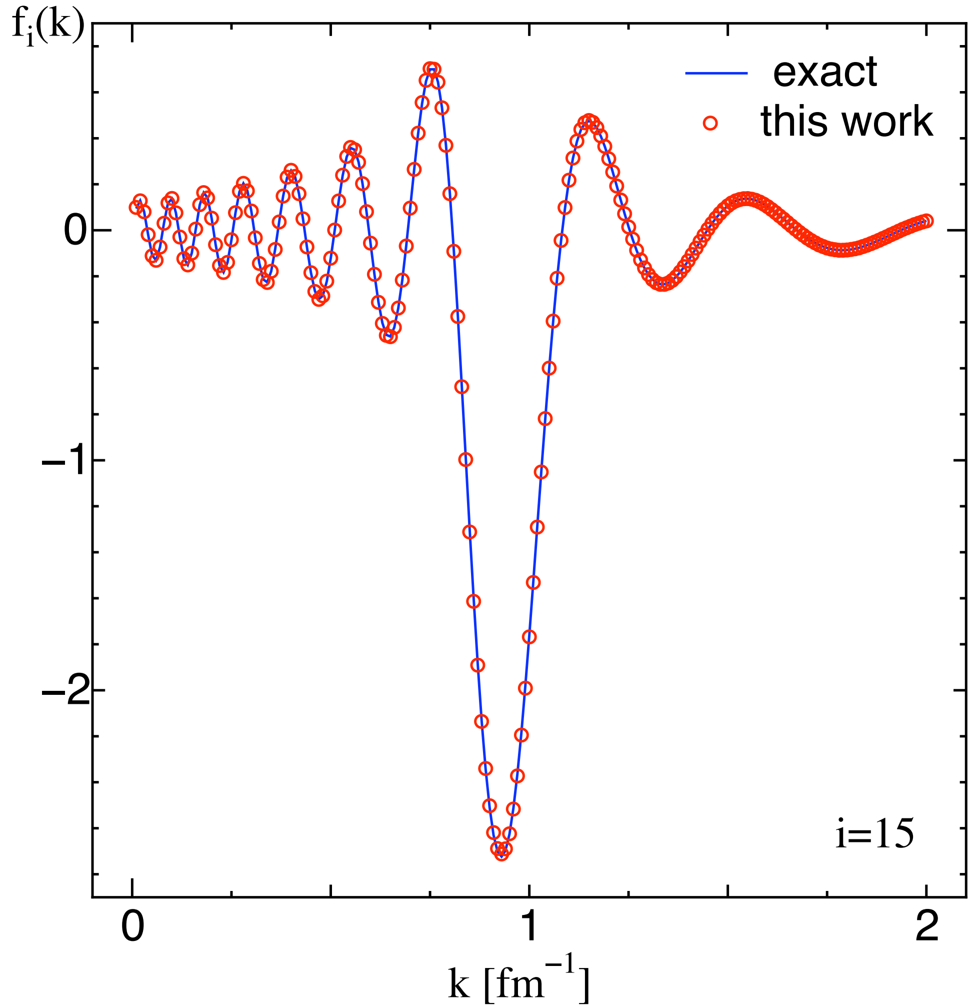


smoothing factor

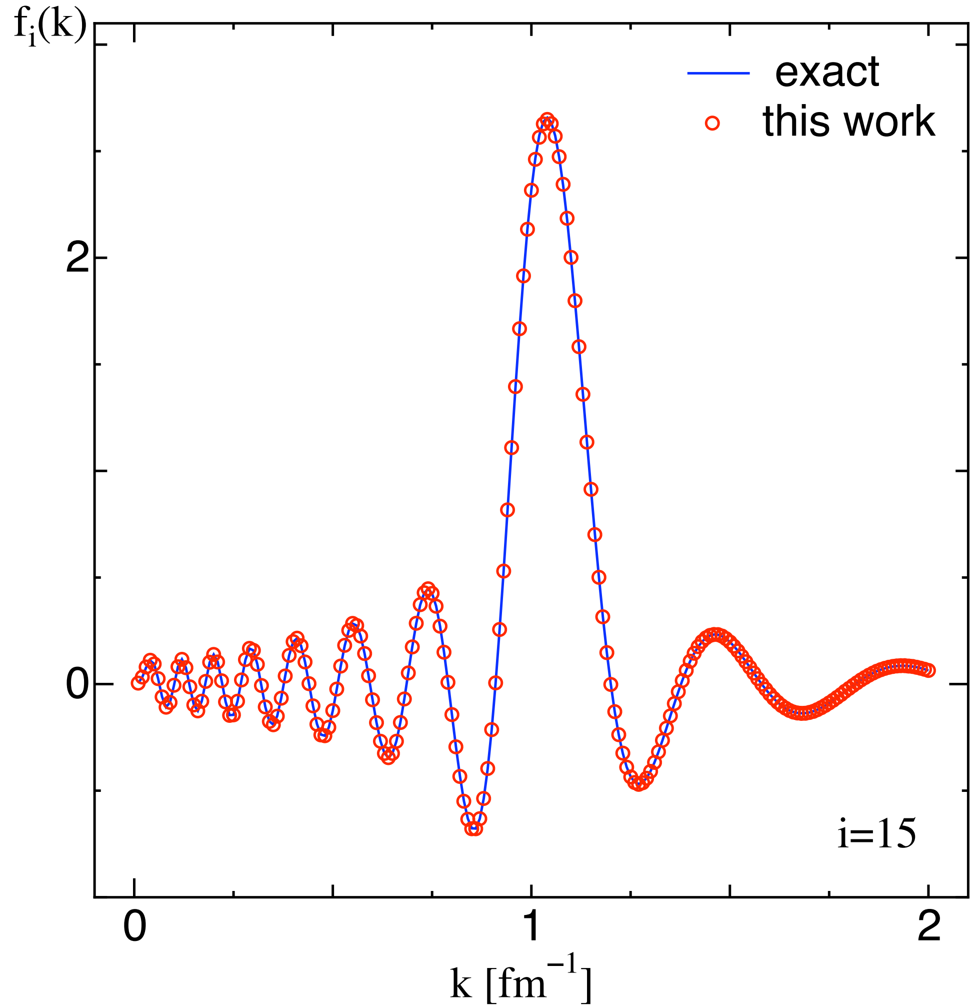


$$f_i(k) = \langle \psi(k,r) | \phi_i(r) \rangle = \int dr \psi(k,r)^* \phi_i(r)$$

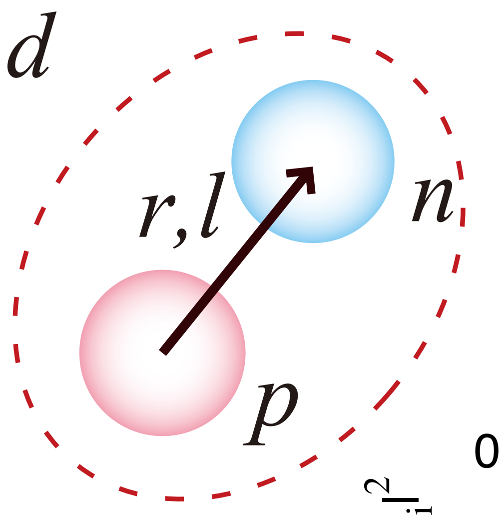
s-state



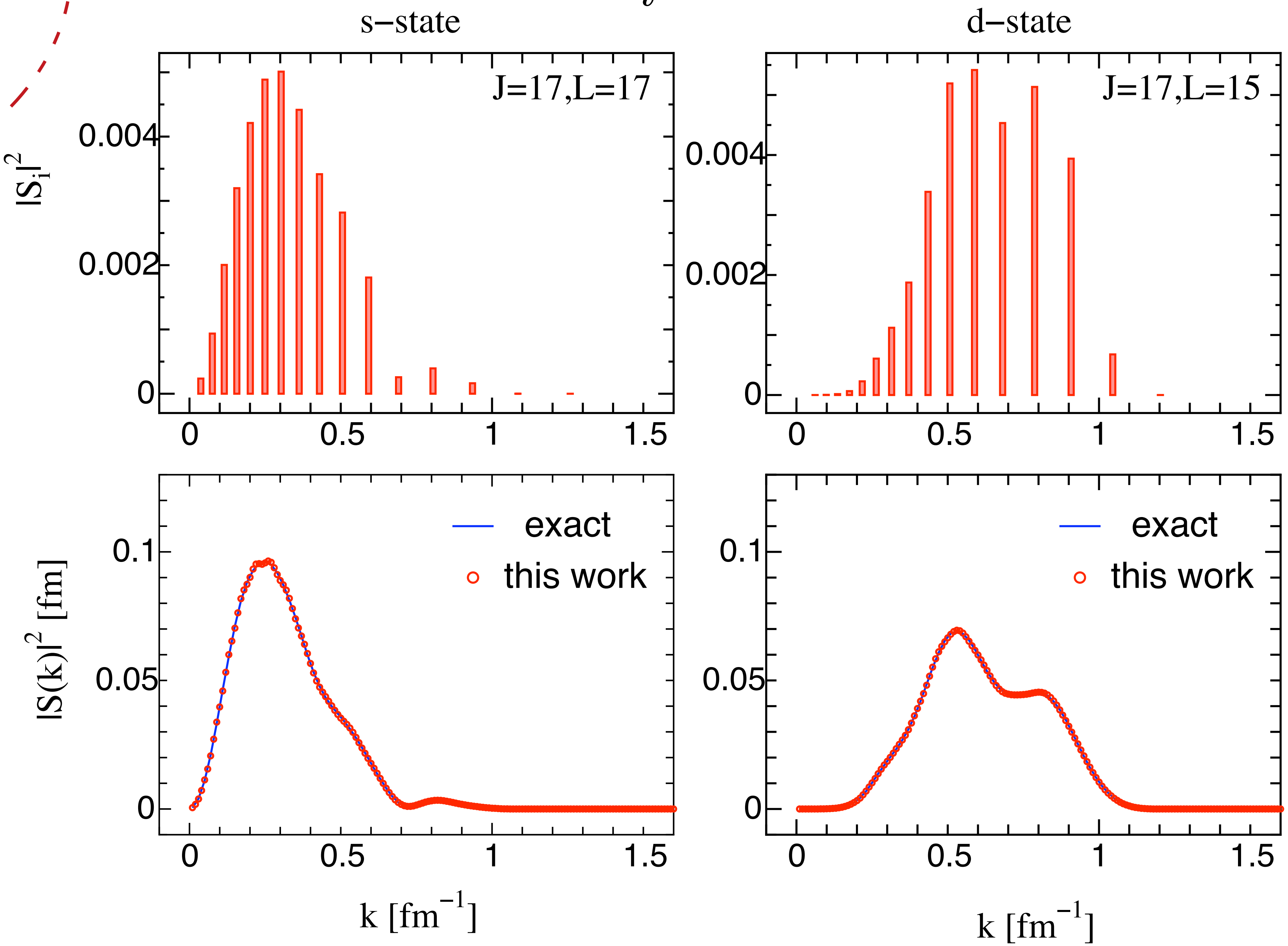
d-state



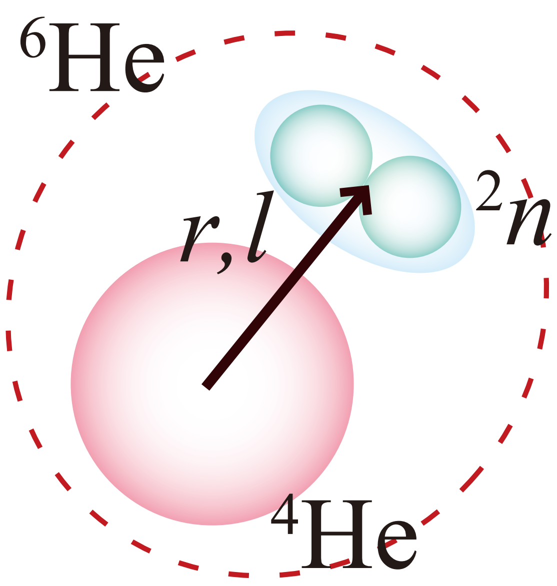
breakup S-matrix



$$S(k) = \sum_i f_i(k)\hat{S}_i$$

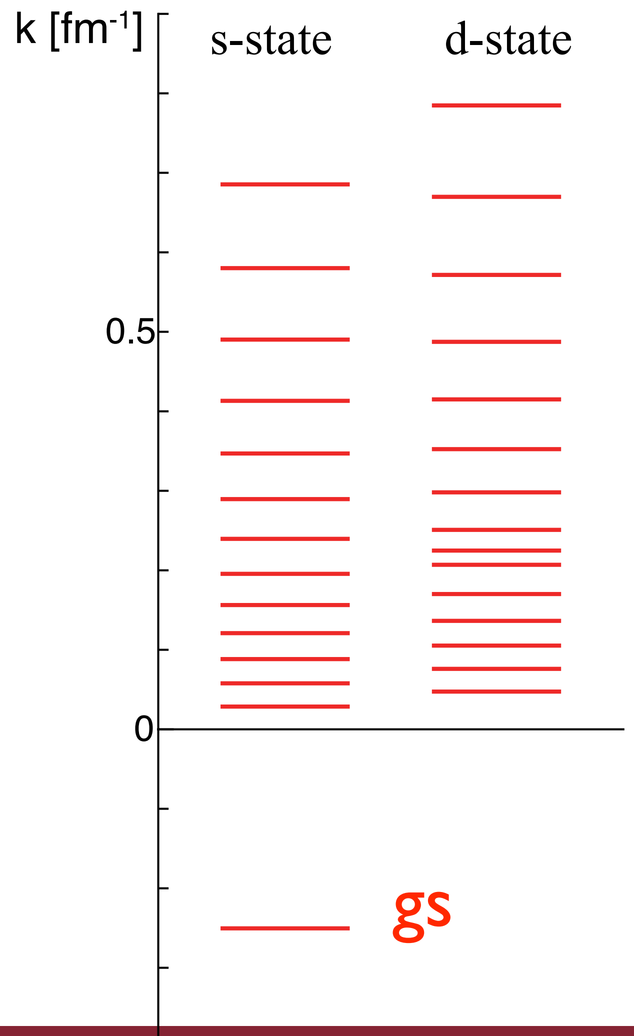


^{6}He (di-neutron model)

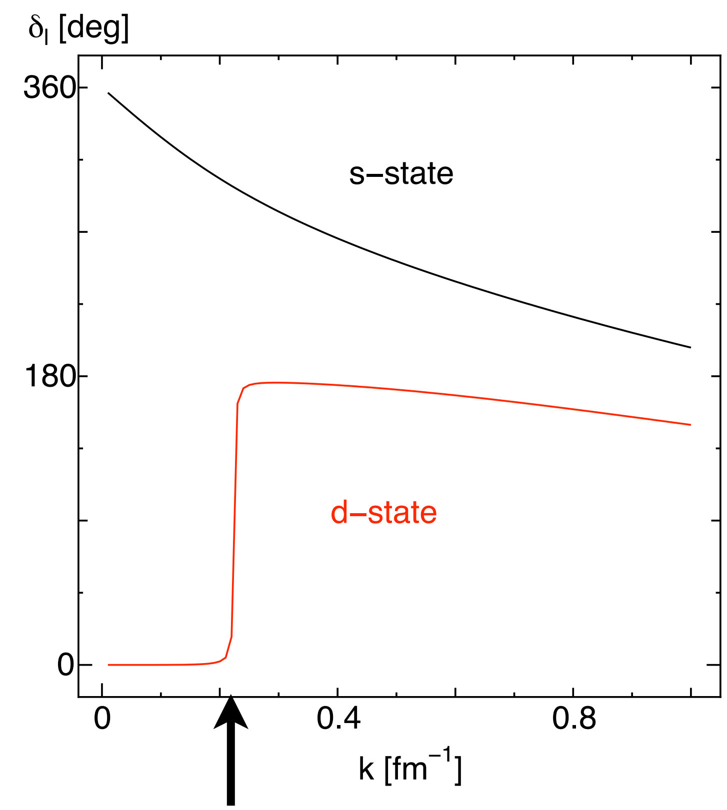


- smoothing factor
- breakup S-matrix

momentum level

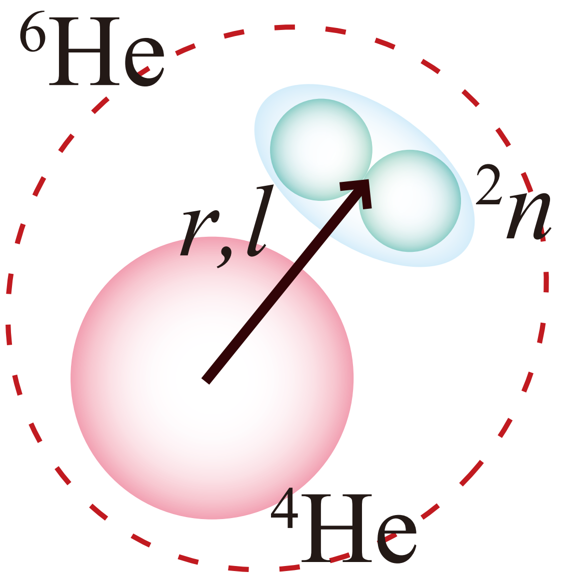


phase shift

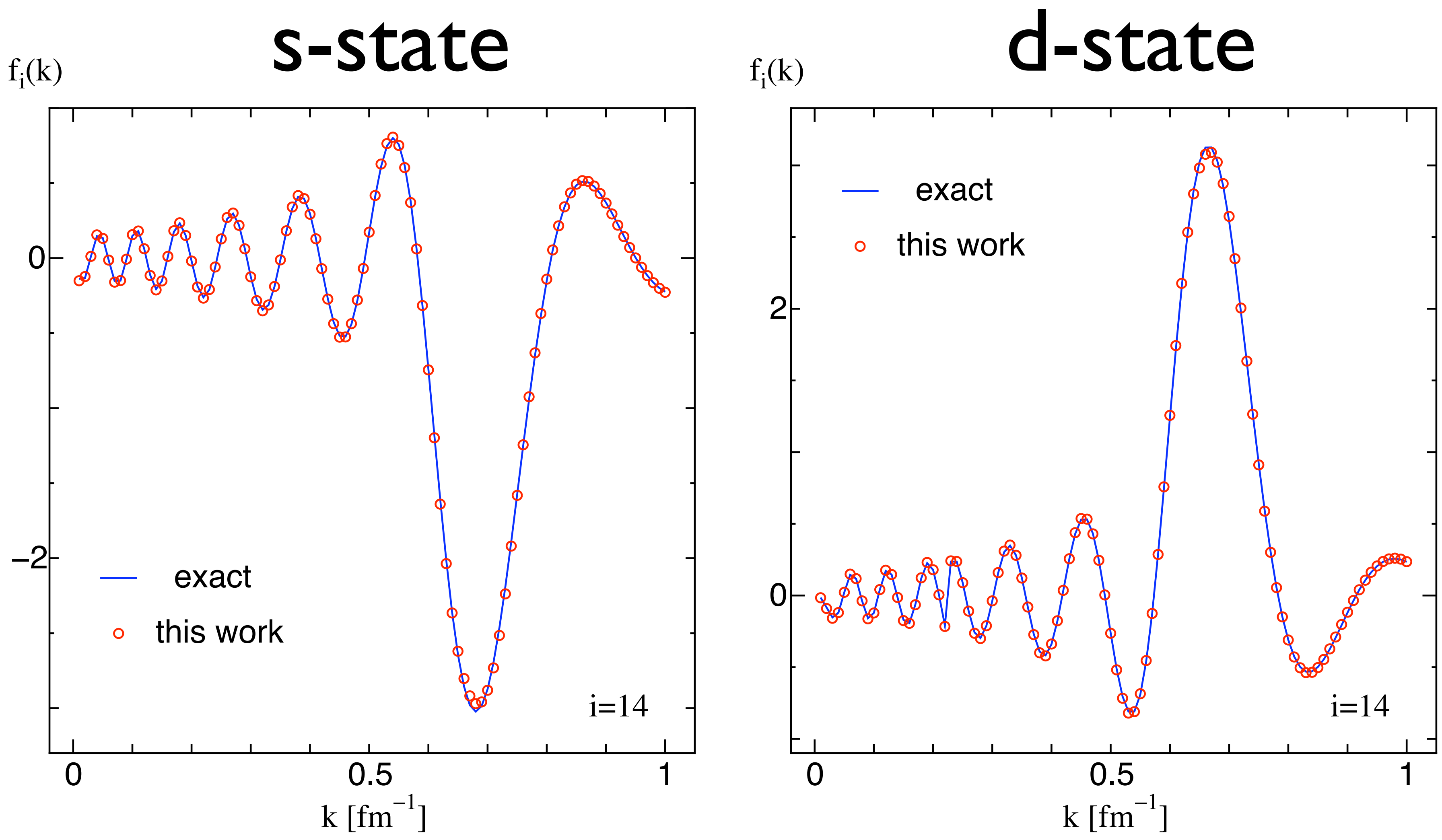


resonance energy

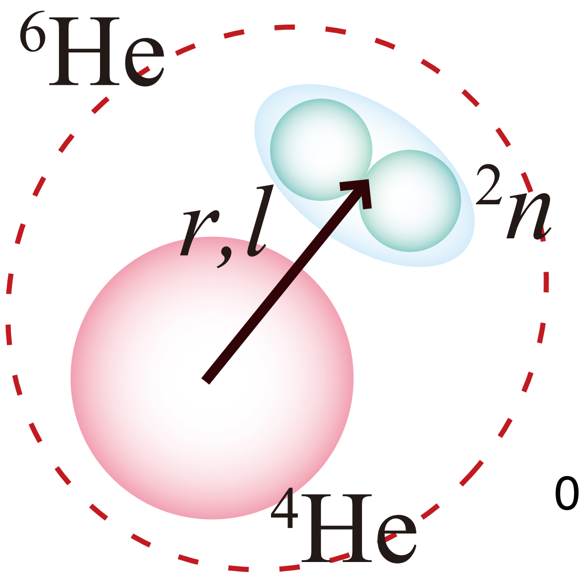
smoothing factor



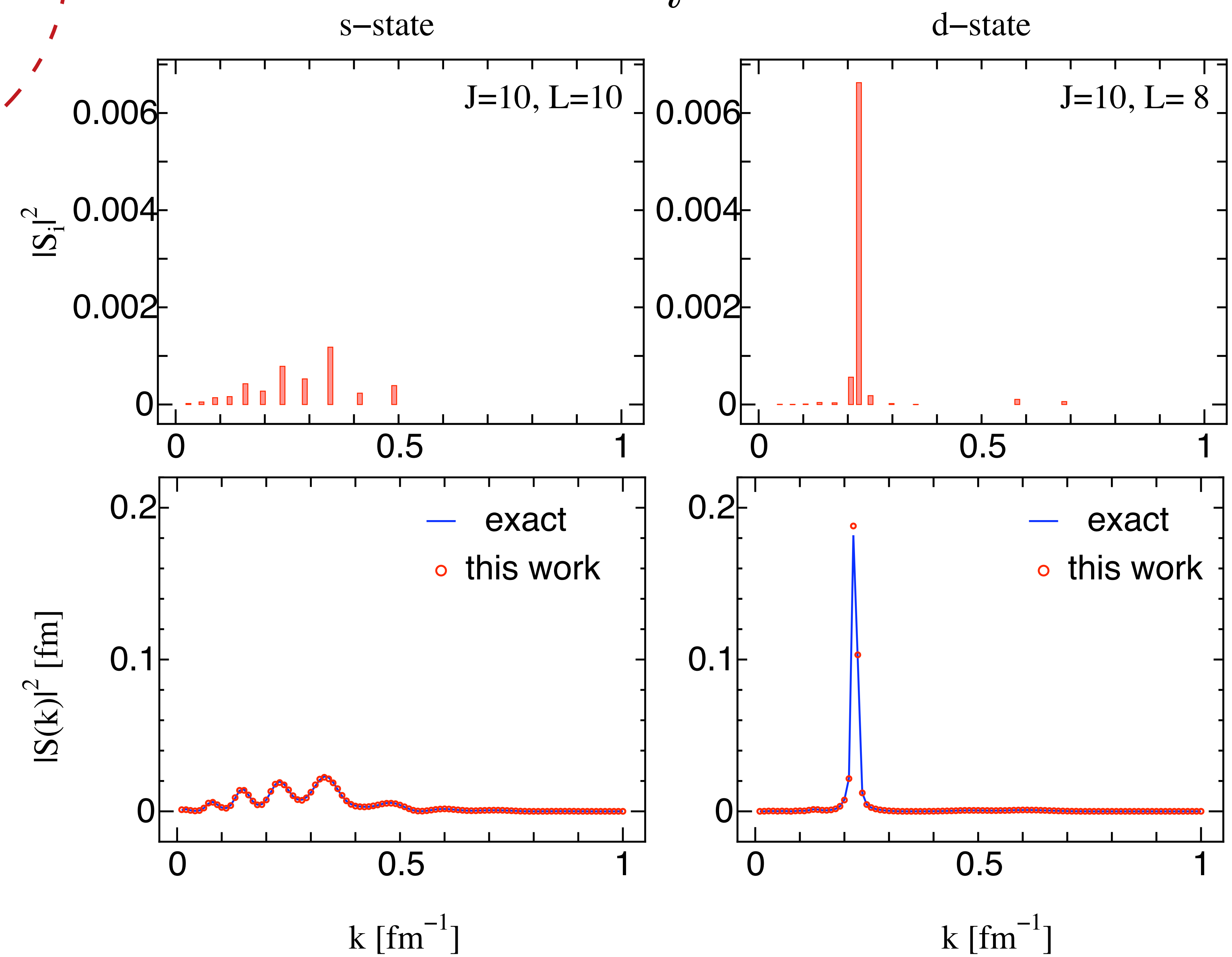
$$f_i(k) = \langle \psi(k,r)|\phi_i(r)\rangle = \int dr \psi(k,r)^* \phi_i(r)$$



breakup S-matrix



$$S(k) = \sum_i f_i(k)\hat{S}_i$$



Three-body case

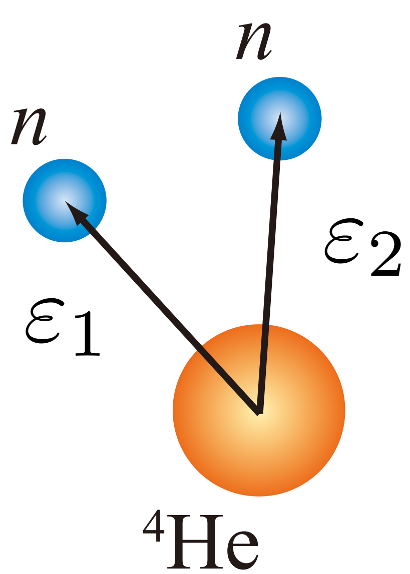
$$\frac{d^2B(E1)}{d\varepsilon_1 d\varepsilon_2} \propto \quad |\langle\psi(\boldsymbol{k},\boldsymbol{K})|\mathcal{O}(E1)|\psi_{gs}\rangle|^2$$

$$\epsilon_k = \frac{\hbar^2 k^2}{2\mu_r}, \qquad \epsilon_K = \frac{\hbar^2 K^2}{2\mu_R}$$

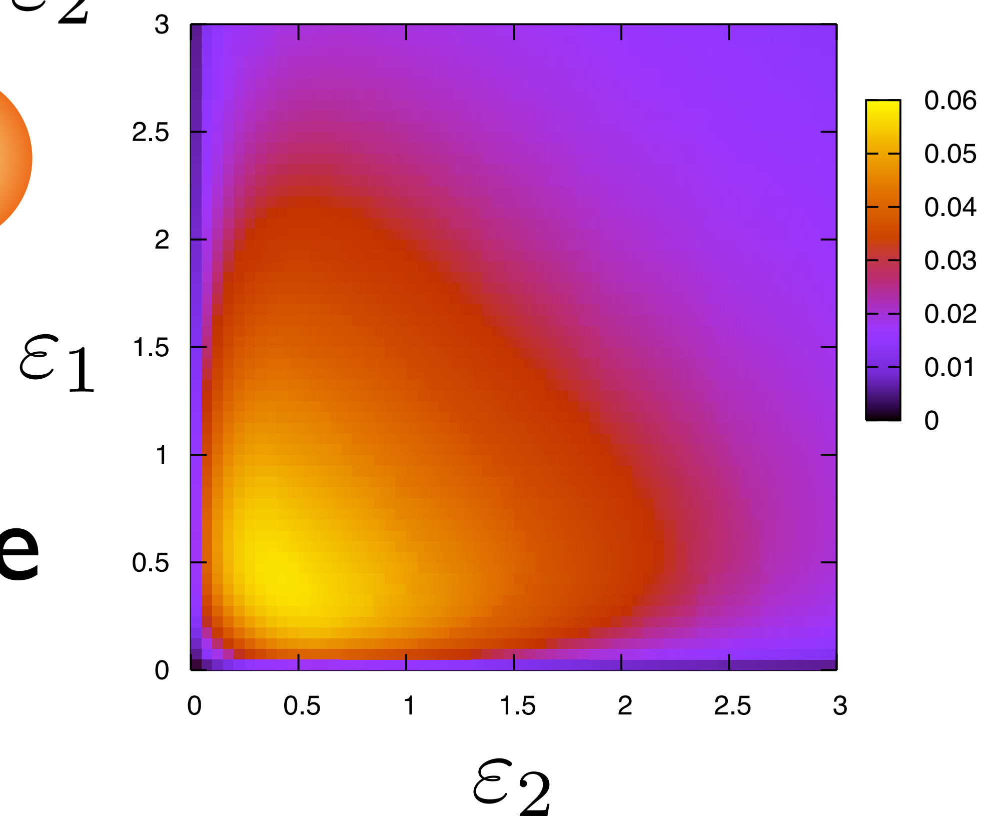
$$= |\sum_i \underline{\langle\psi(\boldsymbol{k},\boldsymbol{K})|\phi_i\rangle}\langle\phi_i|\mathcal{O}(E1)|\psi_{gs}\rangle|^2$$

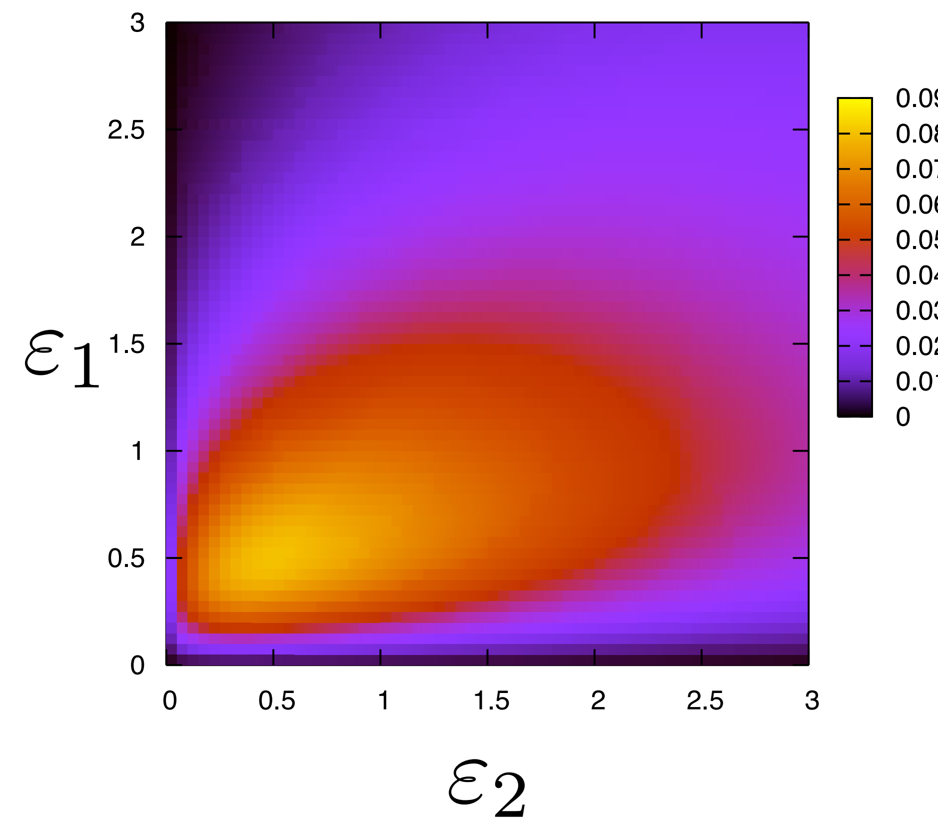
smoothing factor

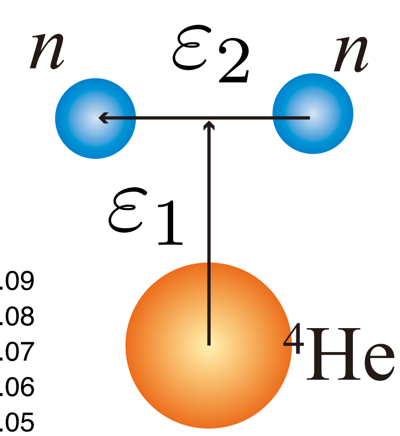
$$\langle\phi_i|\psi\rangle = \langle\phi_i|\psi_0\rangle + \sum_{j,k}\langle\phi_i|\frac{1}{\varepsilon - h_0}|\phi_j\rangle\langle\phi_j|V|\phi_k\rangle\langle\phi_k|\psi\rangle$$



Plane wave part only







V-type

T-type

Three-body case

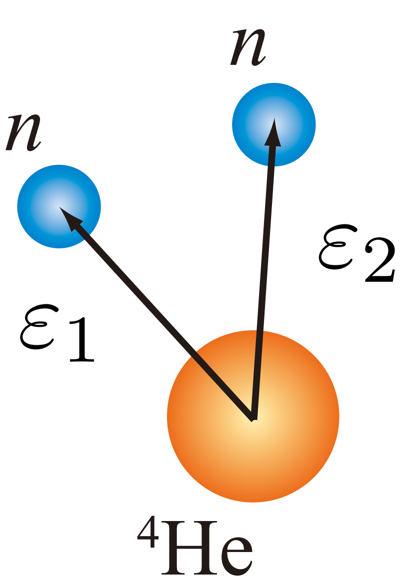
$$\frac{d^2B(E1)}{d\varepsilon_1 d\varepsilon_2} \propto \quad |\langle\psi(\boldsymbol{k},\boldsymbol{K})|\mathcal{O}(E1)|\psi_{gs}\rangle|^2$$

$$= |\sum_i \underline{\langle\psi(\boldsymbol{k},\boldsymbol{K})|\phi_i\rangle}\langle\phi_i|\mathcal{O}(E1)|\psi_{gs}\rangle|^2$$

smoothing factor

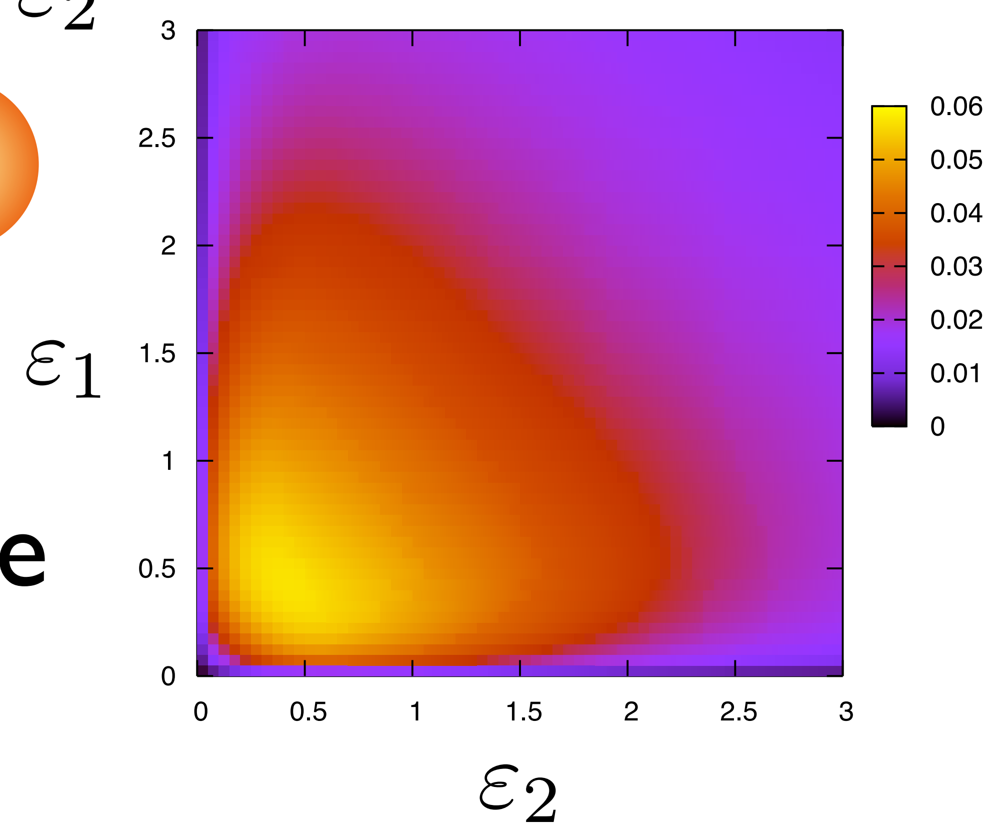
$$\epsilon_k = \frac{\hbar^2 k^2}{2\mu_r}, \qquad \epsilon_K = \frac{\hbar^2 K^2}{2\mu_R}$$

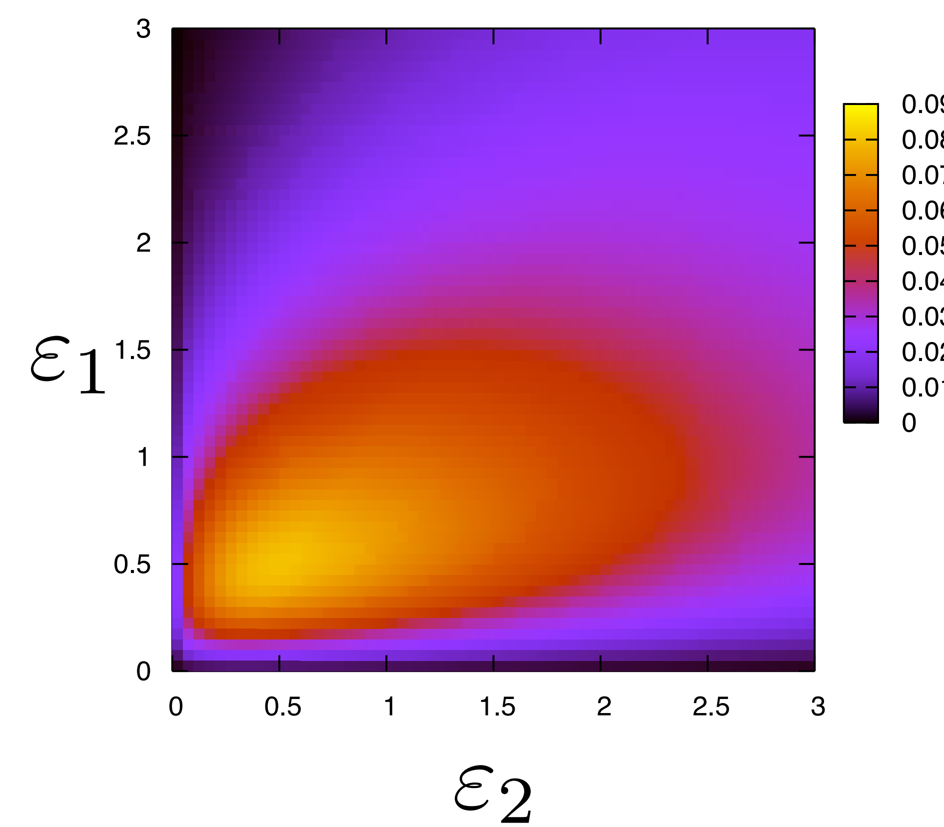
$$\langle\phi_i|\psi\rangle = \langle\phi_i|\psi_0\rangle + \sum_{j,k}\langle\phi_i|\frac{1}{\varepsilon - h_0}|\phi_j\rangle\langle\phi_j|V|\phi_k\rangle\langle\phi_k|\psi\rangle$$

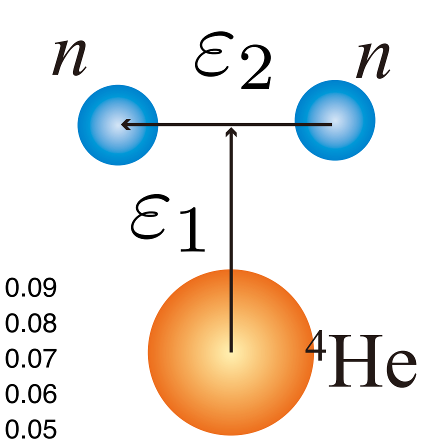


V-type

Plane wave part only







T-type

Summary

- In order to analyze four-body breakup reaction, we have to make breakup S-matrix element as a smooth function of momentum (energy).
- For four-body CDCC (three-body projectile), we cannot obtain smoothing factor by a straightforward way.
  Because it is hard to obtain three-body continuum wave function.
- We propose the method which doesn't need three-body wave function. --Lippmann-Schwinger eq. with model space approx.
  Using this method, we can get smoothing factor directly.
- Checked the validity of new method
  -- two-body system; *d*, ^{6}He (di-neutron model)
- New method can be applied to three-body projectile within four-body CDCC.