Four-body CDCC calculations applied to the scattering of Borromean nuclei

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Complexo Interdisciplinar

Collaboration

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Outline

- Motivation
  - Borromean nuclei
- Discretization methods based on Hyperspherical Harmonics (HH)
  - Transformed Harmonic Oscillator (THO)
  - Bin
- Four-body Contiuum Discretized Coupled Channels (CDCC)
- Application to ^{6}He+target
- Summary and conclusions

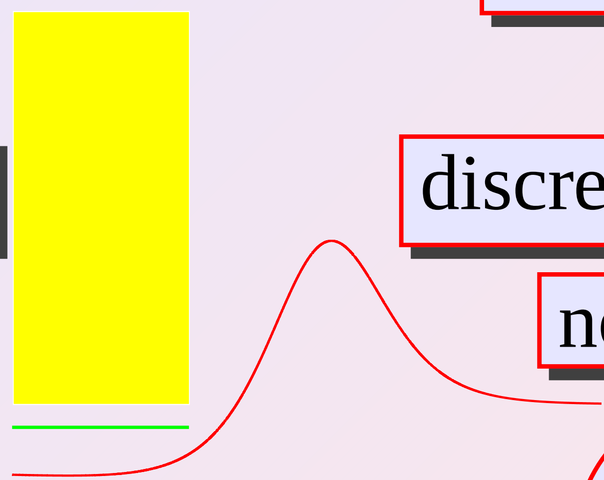
Motivation: General scheme

quantum system

↙ ↘

unbound states ⇓ continuum spectrum, non normalizable

bound states ⇓ discrete spectrum, normalizable



▼ weakly bound system

discretization method

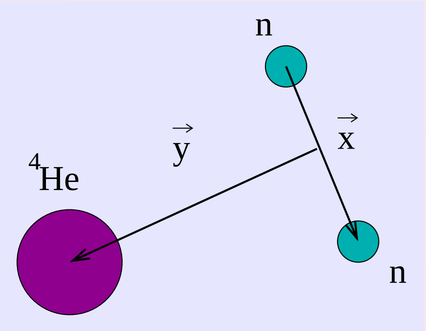
▼ ↪ strongly bound system

Three-body discretization methods

- HH method: The states of the system are expanded in Hyperspherical Harmonics

$$\Psi_{j\mu n}(\rho,\Omega) = \sum_{\beta} R_{\beta jn}(\rho) \sum_{\nu\iota} \langle j_{ab}\nu I\iota | j\mu\rangle \kappa_I^{\iota} \sum_{m\sigma} \langle lmS_x\sigma | j_{ab}\nu\rangle \Upsilon_{Klm}^{l_x l_y}(\Omega) \chi_{S_x}^{\sigma}$$

$$\Omega \equiv \{\alpha, \widehat{x}, \widehat{y}\}$$
$$\beta \equiv \{\mathrm{K}, l_x, l_y, l, S_x, j_{ab}\}$$



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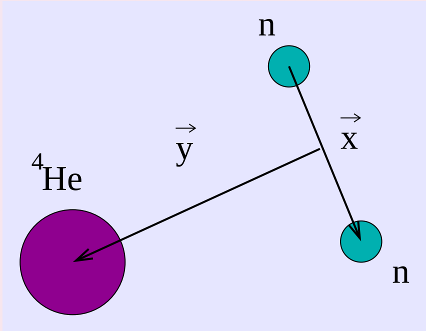

- The hyperradial functions $\{R_{\beta jn}\}$ can be constructed by different discretization methods

THO method: 2-body system

☛ M.V. Stoitsov and I. Zh. Petkov, Ann. Phys., **184**, 121 (1988)

central potential

$$s(r)$$

$$\varphi_B(r) \quad \rightsquigarrow \quad \phi^{HO}_{0l_B}(s)$$

$$\Updownarrow$$

$$\int_0^r |\varphi_B(r')|^2 dr' = \int_0^s |\phi^{HO}_{0l_B}(s')|^2 ds'$$

$$\Downarrow$$

THO basis

$$\psi^{THO}_{nl}(r) = \varphi_B(r)\, s(r)^{l-l_B} L_n^{l+1/2}\left(s(r)^2\right)$$

THO method: 3-body system

- $s(\rho)$ is calculated for each channel β included in the bound ground state

$$\int_0^{\rho} d\rho' \rho'^{5/2} |R_{B\beta}(\rho')|^2 = \int_0^{s} ds' s'^{5/2} |R_{0K}^{HO}(s')|^2$$

$$R_{i\beta}^{THO}(\rho) = R_{B\beta}(\rho) L_i^{K+2}\left(s_\beta(\rho)^2\right)$$

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- The Hamiltonian of the system is diagonalized in a finite THO basis with $i = 0, \ldots, n_b$
- Finally the hyperradial functions are obtained as

$$R_{\beta j n}^{THO}(\rho) = \sum_i C_n^{i\beta j} R_{i\beta}^{THO}(\rho)$$

Bin method (I)

- Continuum states can be expanded in HH as

$$\Psi_{\kappa j\mu}(\rho,\Omega,\Omega_\kappa) = \sum_{\beta\beta'} R_{\beta\beta' j}(\kappa\rho)\mathcal{Y}_{\beta j\mu}(\Omega) \times \sum_{m'\sigma'}\langle l'm'S'_x\sigma'|j\mu\rangle \Upsilon^{l'_x l'_y}_{K'l'm'}(\Omega_\kappa)$$

$\{\beta'\}$ incoming; $\{\beta\}$ outgoing; $\kappa = \sqrt{2m\varepsilon}/\hbar$

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$\{\beta'\}$ incoming; $\{\beta\}$ outgoing; $\kappa = \sqrt{2m\varepsilon}/\hbar$

- Bins for each incoming channel are calcuated as

$$R^{bin}_{j\beta\{\beta'\varepsilon_{av}\}}(\rho) = \frac{2}{\sqrt{\pi N_{\beta' j}}} \int_\kappa^{\kappa+\Delta\kappa} d\kappa f_{\beta' j}(\kappa) R_{\beta\beta' j}(\kappa\rho)$$

$$f^{n-r}_{\beta' j}(\kappa) = e^{-i\delta_{\beta' j}(\kappa)} \qquad f^{r}_{\beta' j}(\kappa) = \sin\delta_{\beta' j}(\kappa) e^{-i\delta_{\beta' j}(\kappa)}$$

$$N_{\beta' j} = \int_\kappa^{\kappa+\Delta\kappa} d\kappa |f_{\beta' j}(\kappa)|^2$$

Bin method (II)

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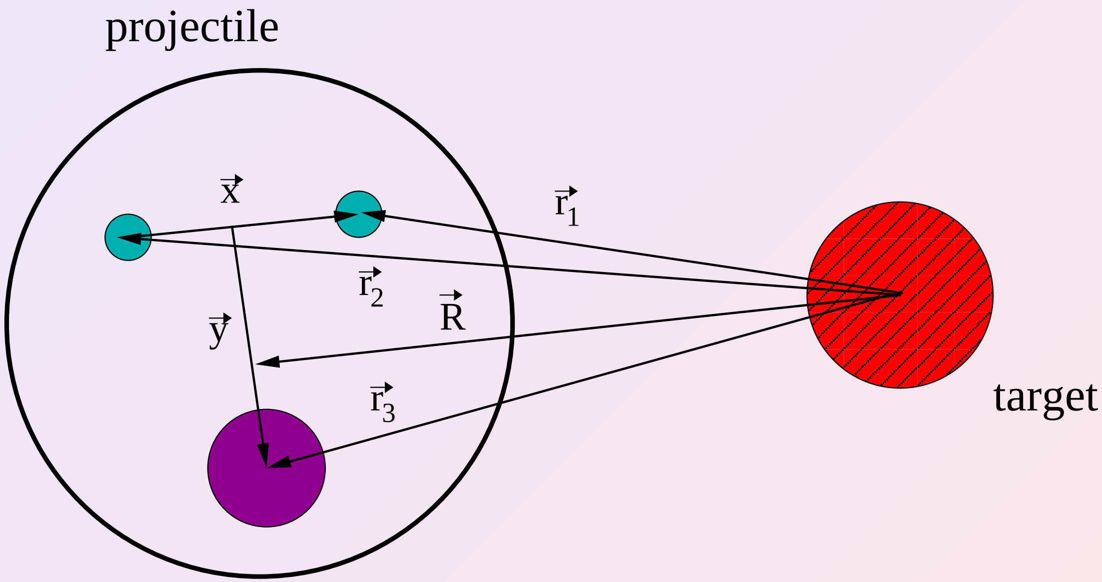
- The inclusion of all incoming channels is computationally unreachable for reaction calculations presented next
- So S-matrix is diagonalized for every ε obtaining the eigenchannels and eigenphases
- Then bins are calcualted for each eigenchannel such as explained before for incoming channels
- Now we include only up to n_{ec} eigenchannels that corresponds to the biggest phase-shifts

4-body CDCC formalism

Coupled channels system

$$\left[-\frac{\hbar^2}{2m_r}\left(\frac{d^2}{dR^2}-\frac{L(L+1)}{R^2}\right)+\varepsilon_{nj}-E\right]f^J_{Lnj}(R)$$
$$+\sum_{L'n'j'} i^{L'-L} V^J_{Lnj,L'n'j'}(R) f^J_{L'n'j'}(R) = 0$$



Coupling potentials

$$V^J_{Lnj,L'n'j'}(R) = \langle LnjJM| \sum_{k=1}^{3} \widehat{V}_{kt}(\vec{r}_k)|L'n'j'JM\rangle$$

where

$$\Phi^{JM}_{Lnj}(\widehat{R},\vec{x},\vec{y}) = \sum_{\mu M_L} \psi_{j\mu n}(\vec{x},\vec{y})\langle LM_L j\mu|JM\rangle Y_{LM_L}(\widehat{R}$$

multipolar expansion

$$V^J_{Lnj,L'n'j'}(R) = \sum_Q (-1)^{J-j}\hat{L}\hat{L}' \begin{pmatrix} L & Q & L' \\ 0 & 0 & 0 \end{pmatrix} \times W(LL'jj',QJ)F^Q_{nj,n'j'}(R)$$

Form factors

$$
\begin{aligned}
F^{Q}_{nj,n'j'}(R) &= (-1)^{Q+2j-j'}\hat{j}\hat{j}'(2Q+1) \\
&\times \sum_{\beta\beta'} \sum_{k=1}^{3} \sum_{\beta_k\beta'_k} N_{\beta\beta_k} N_{\beta'\beta'_k} \\
&\times (-1)^{l_{xk}+S_{xk}+j'_{abk}-j_{abk}-I_k} \delta_{l_{xk}l'_{xk}} \delta_{S_{xk}S'_{xk}} \\
&\times \hat{l}_{yk}\hat{l}'_{yk}\hat{l}_k\hat{l}'_k\hat{j}_{abk}\hat{j}'_{abk} \begin{pmatrix} l_{yk} & Q & l'_{yk} \\ 0 & 0 & 0 \end{pmatrix} \\
&\times W(l_k l'_k l_{yk} l'_{yk}; Q l_{xk}) W(j_{abk} j'_{abk} l_k l'_k; Q S_{xk}) \\
&\times W(j j' j_{abk} j'_{abk}; Q I_k) \int\int (\sin\alpha_k)^2 (\cos\alpha_k)^2 \rho^5 d\alpha_k\, d\rho \\
&\times R_{\beta j n}(\rho) \varphi^{l_{xk} l_{yk}}_{K_k}(\alpha_k) \mathcal{V}^k_Q(R, y_k) \varphi^{l_{xk} l'_{yk}}_{K'_k}(\alpha_k) R_{\beta' j' n'}(\rho)
\end{aligned}
$$

^{6}He Hamiltonian

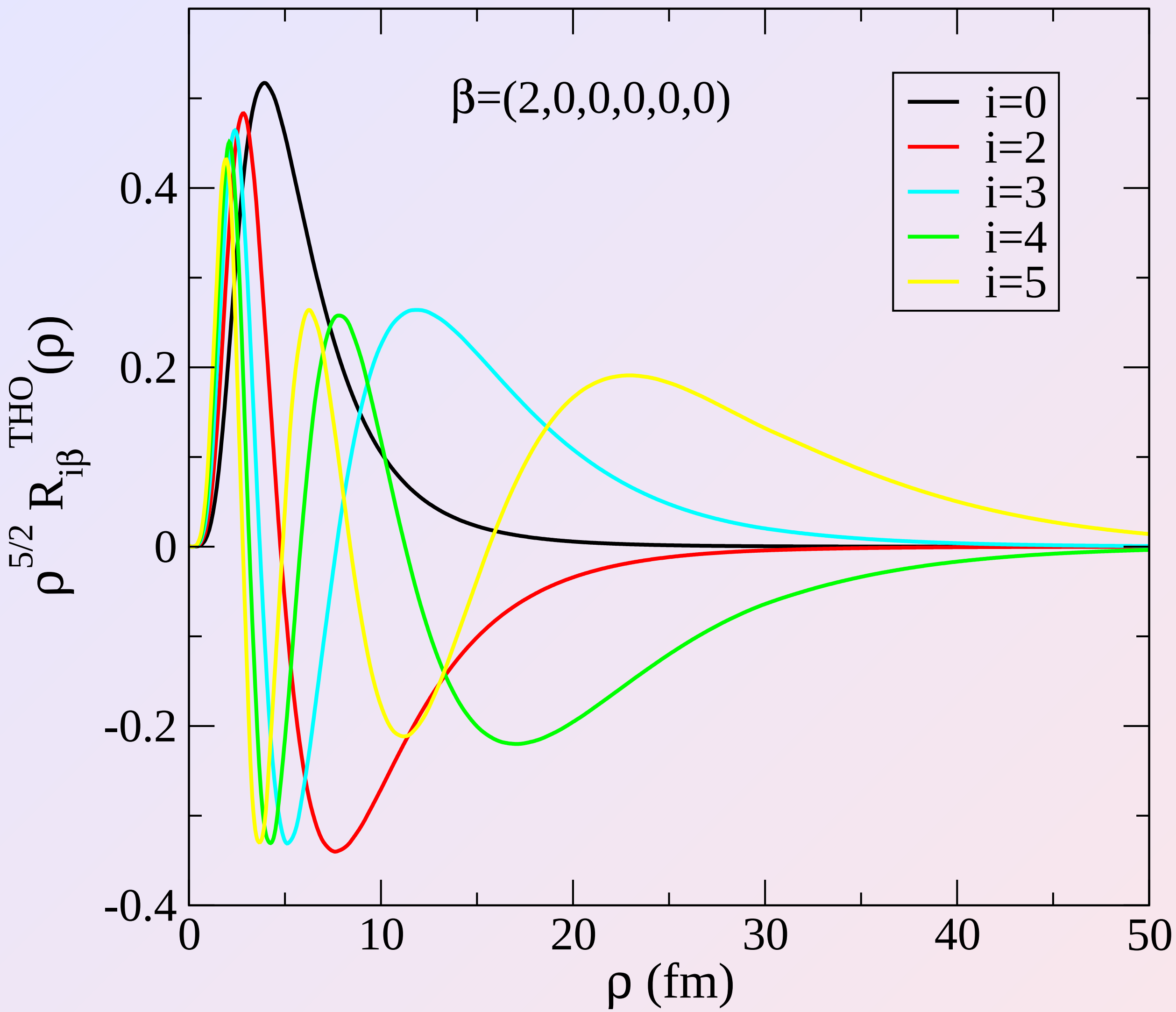
$$\widehat{H}(\rho,\Omega) = \widehat{T}(\rho,\Omega) + \widehat{V}(\rho,\Omega)$$

$$V = V_{n\alpha} + V_{n\alpha} + V_{nn} + V_{nn\alpha}$$

- $n+\alpha$ $V_{n\alpha} = V_c + V_{SO}$
  V_c, V_{SO}: Woods-Saxon
- GPT $n+n$ $V_{nn} = V_c + V_{SO} + V_t$
  V_c, V_t, V_{SO}: Gaussian
- $n+n+\alpha$: power $V_{pow} = \frac{a}{[1+(r/b)^c]}$

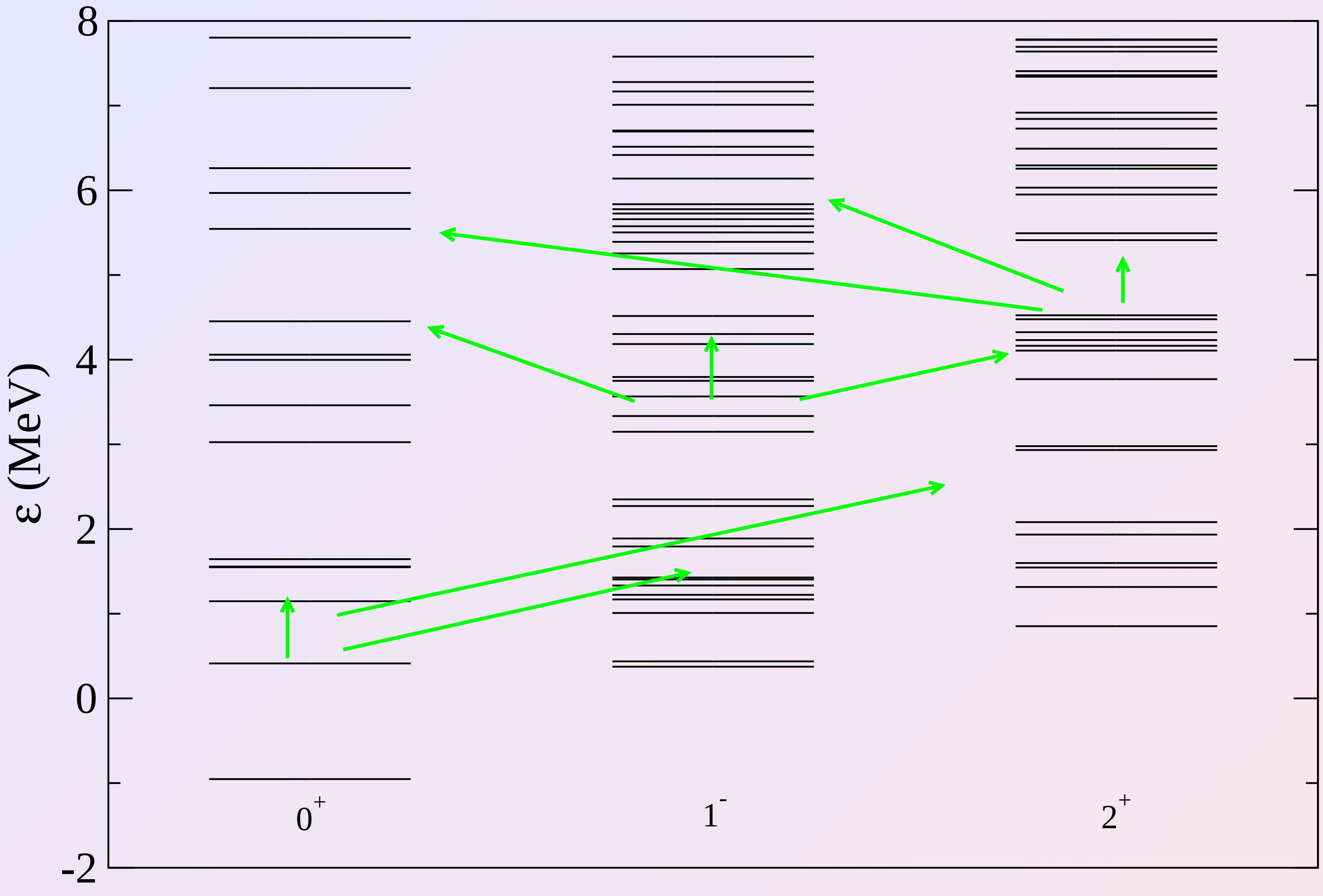
Pauli forbidden states: repulsive V_c for s-waves

THO basis



$K_{max} = 8$

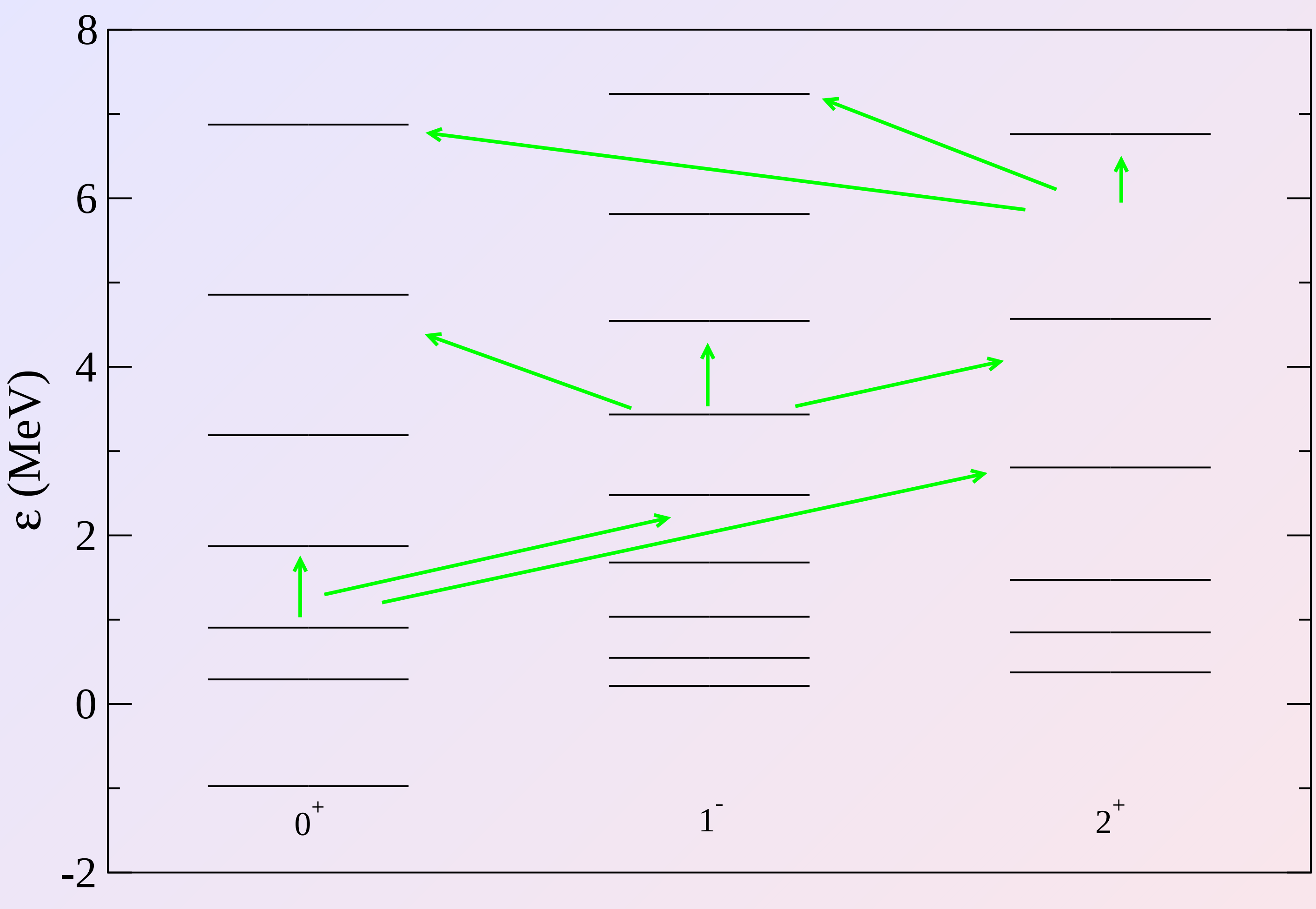
THO: Energy spectrum



$K_{max} = 8$

$n_b = 4$

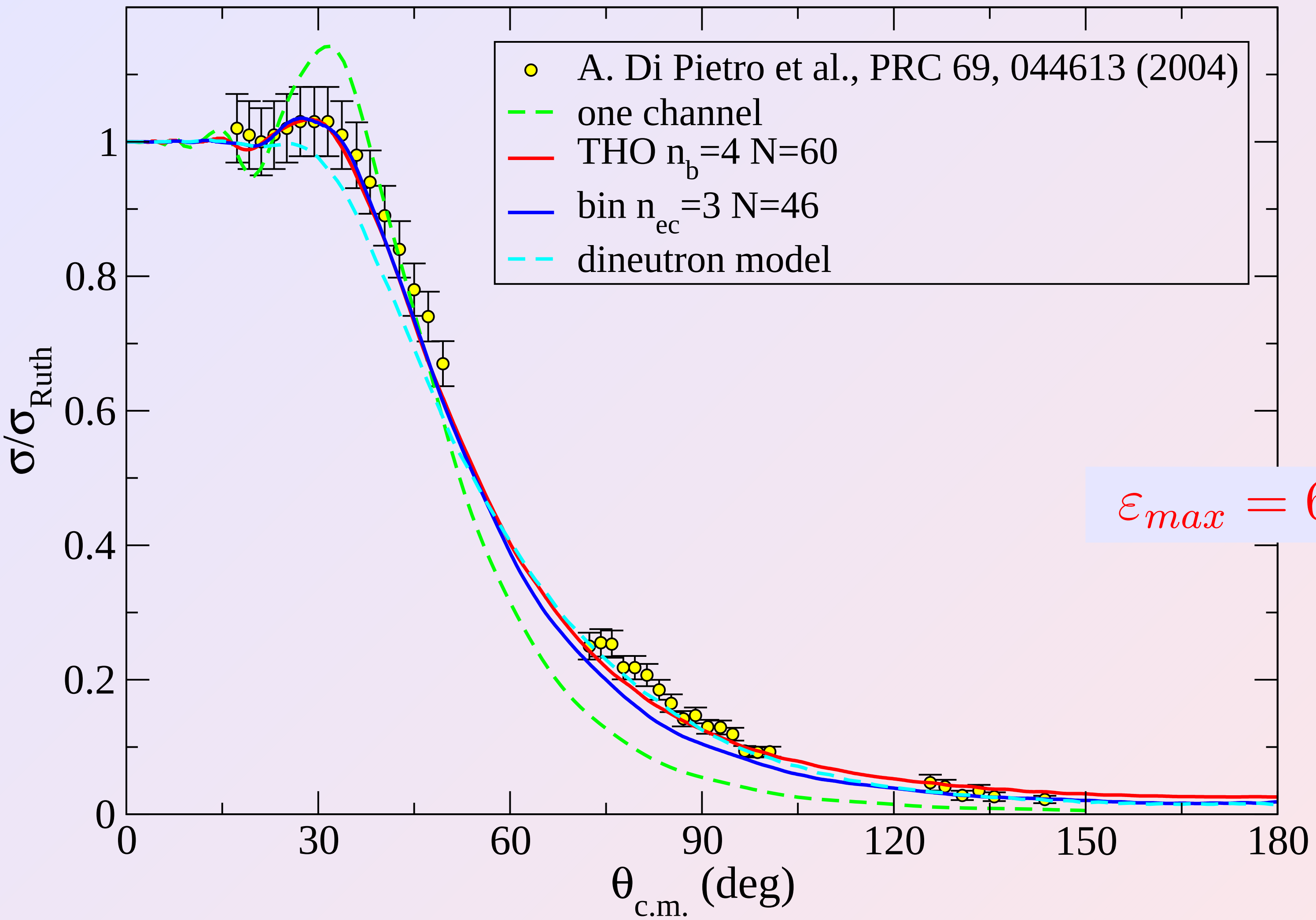
Bin: Energy spectrum



$K_{max} = 8$

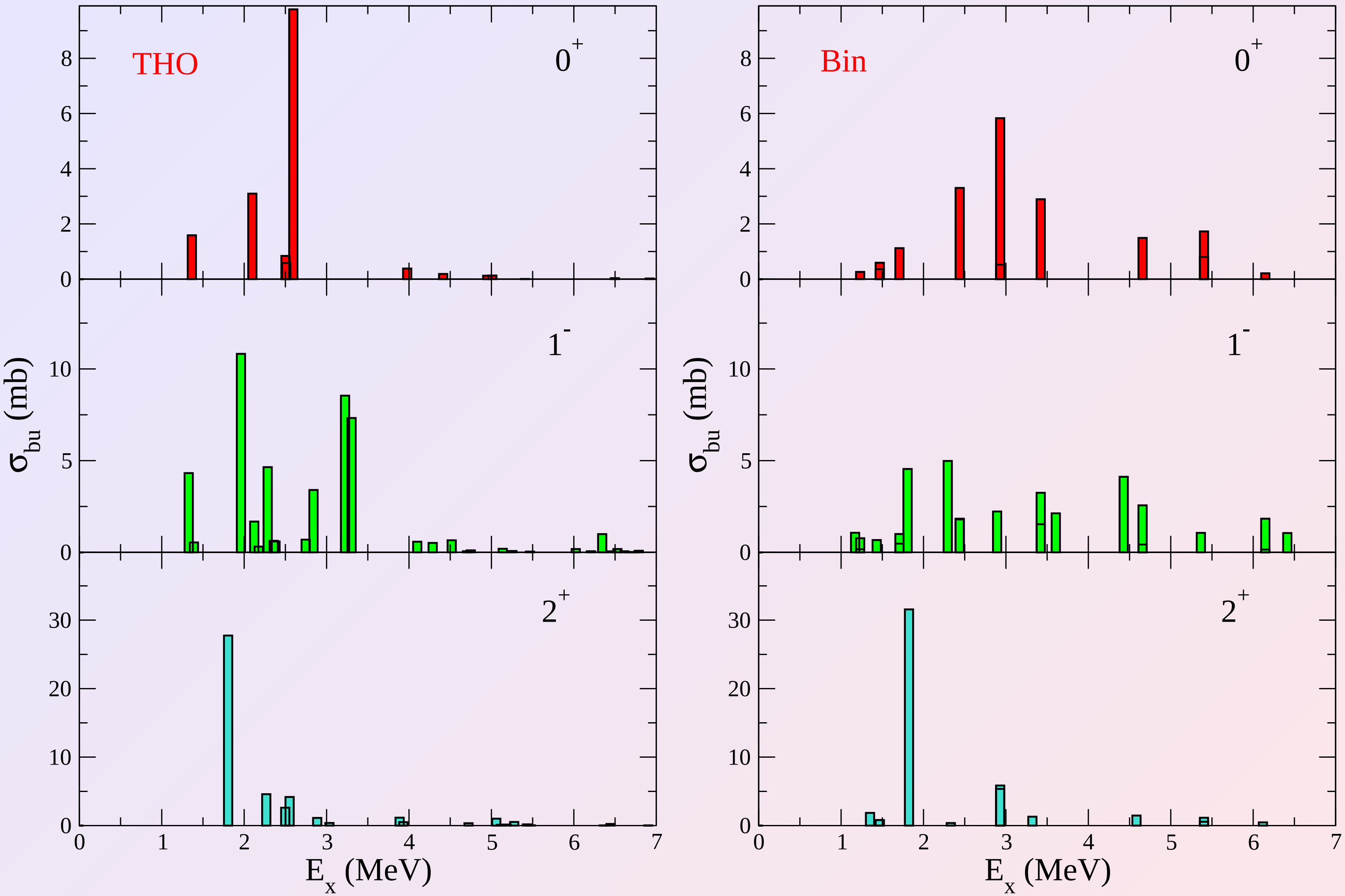
$1ec$

^{6}He+^{64}Zn@13.6MeV: elastic

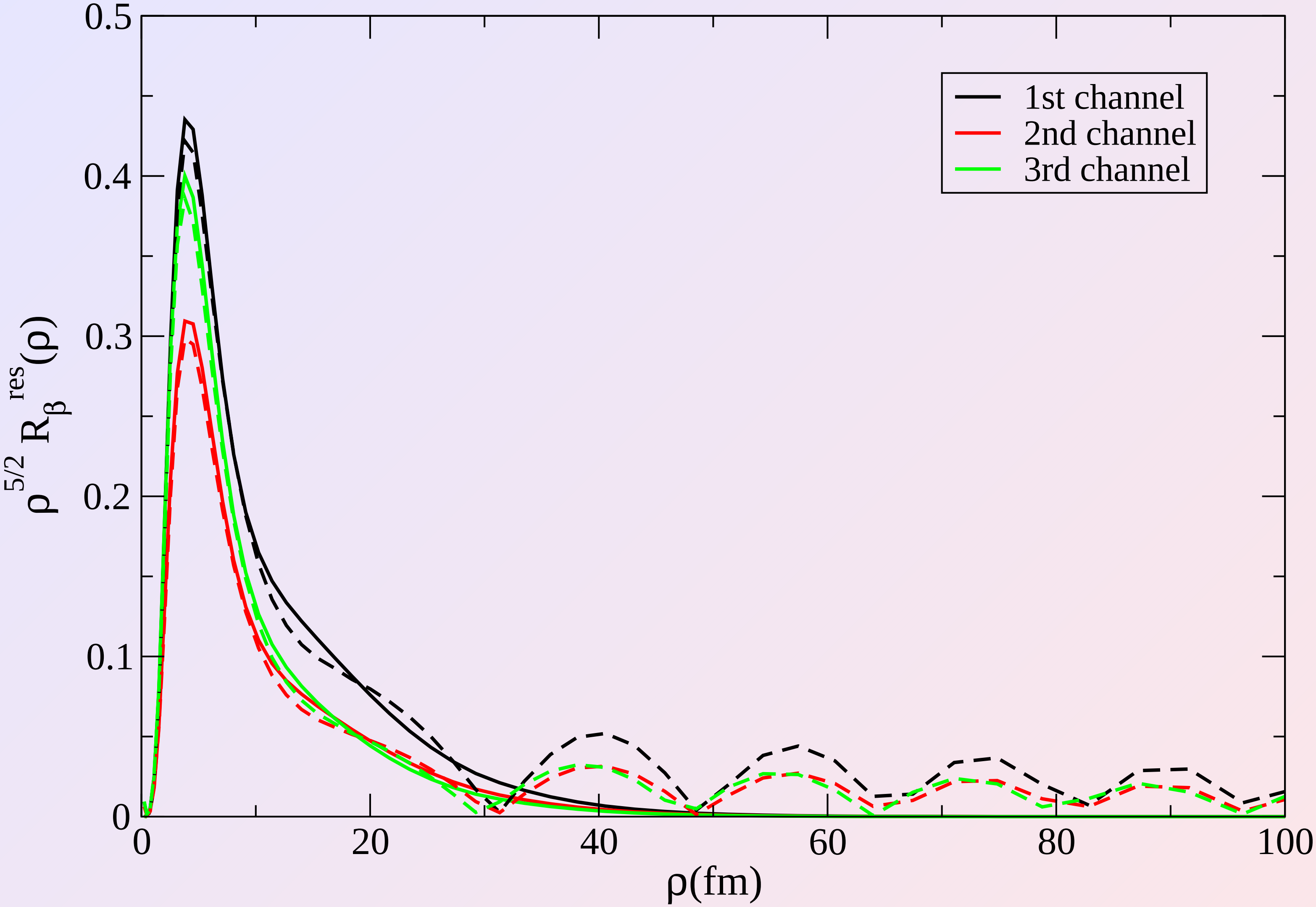


$\varepsilon_{max} = 6$ MeV

$^{6}He+^{64}Zn$@13.6MeV: breakup

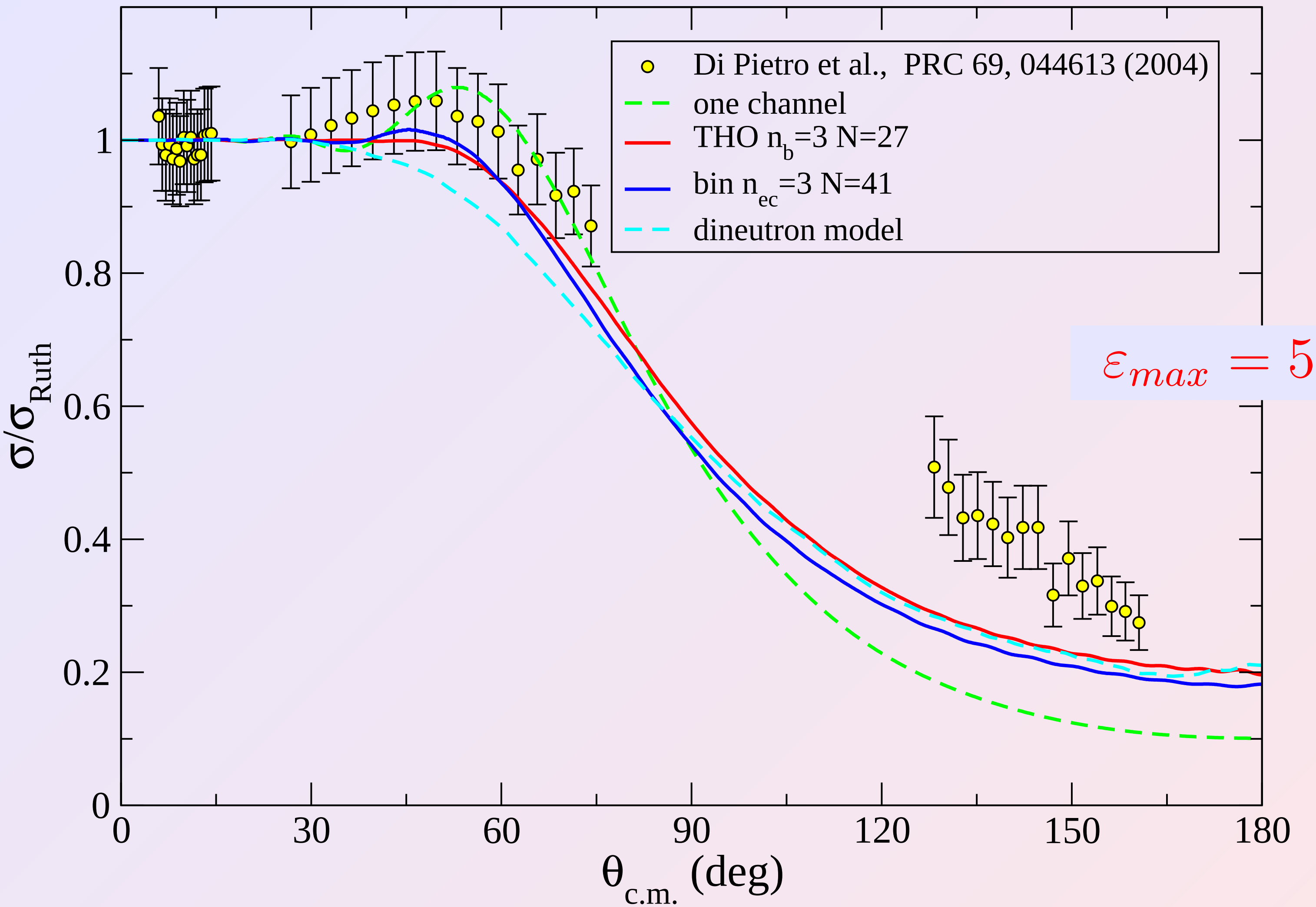


2^+ resonance



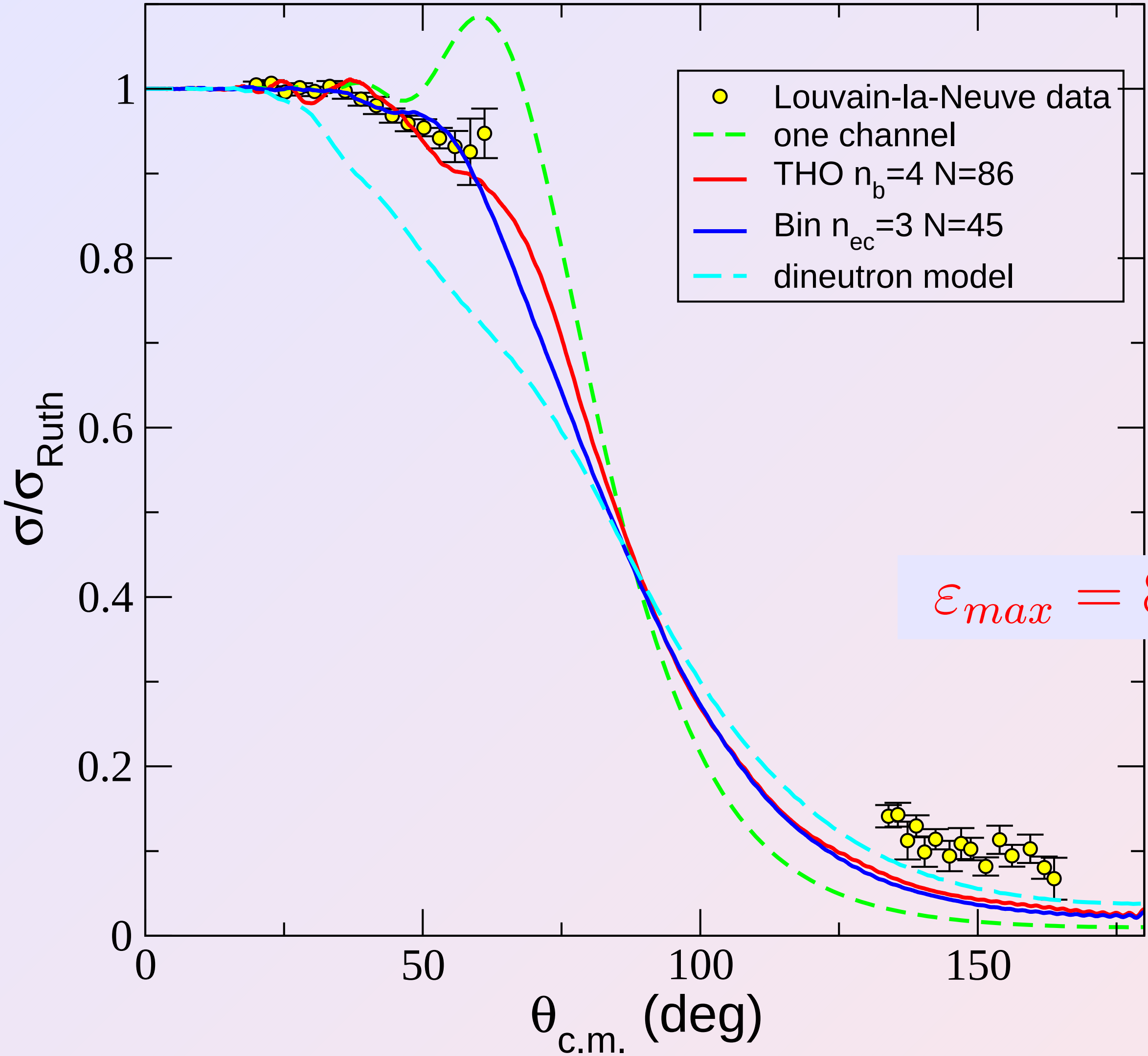
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
ρ(fm)
$\rho^{5/2} R_\beta^{res}(\rho)$
1st channel
2nd channel
3rd channel

^{6}He+^{64}Zn@10MeV: elastic



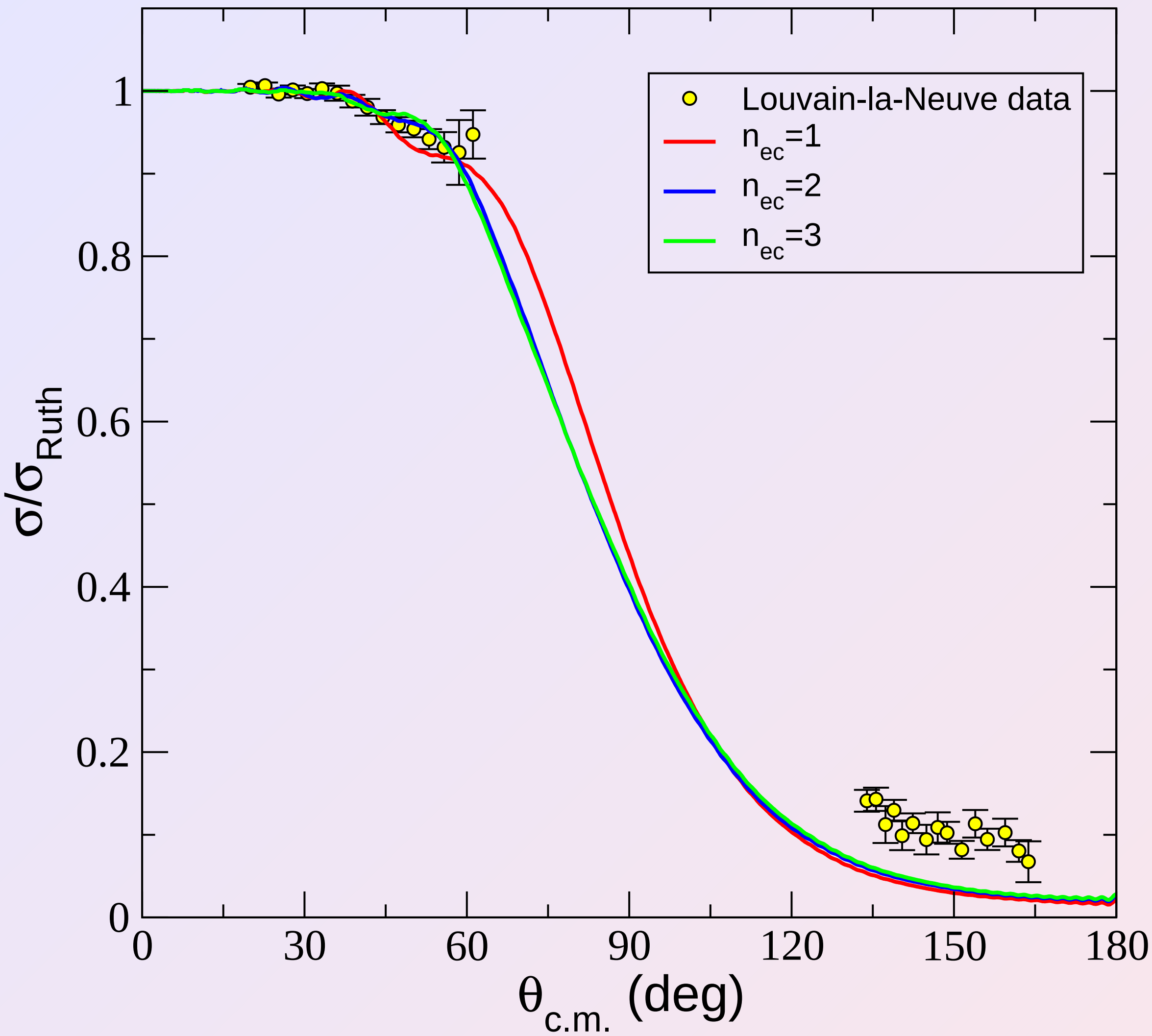
$\varepsilon_{max} = 5$ MeV

^{6}He+^{208}Pb@22MeV: elastic

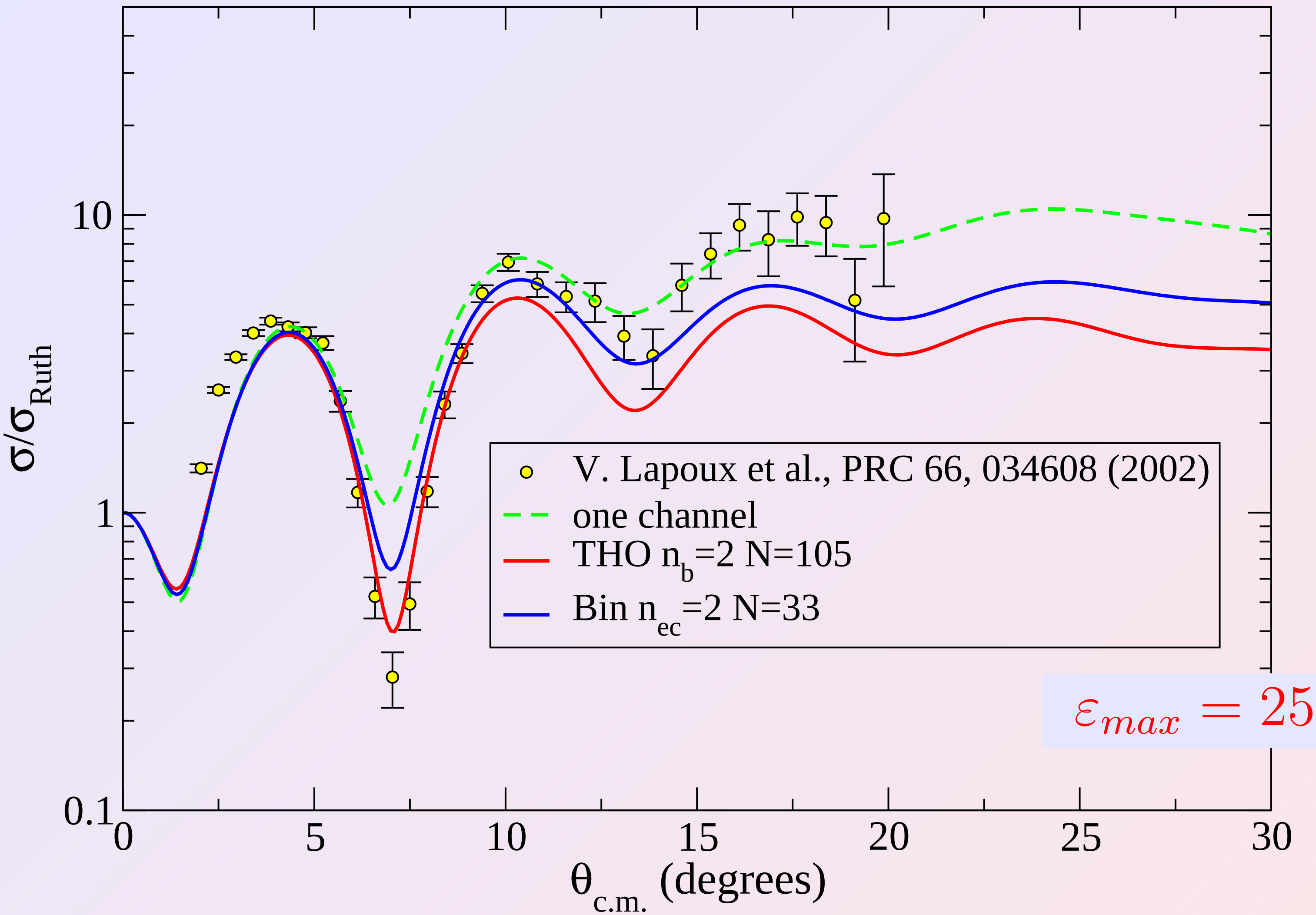


$\varepsilon_{max} = 8$ MeV

$^{6}He+^{208}Pb$@22MeV: convergence



^{6}He+^{12}C@229.8MeV: elastic



$\varepsilon_{max} = 25$ MeV

Summary and conclusions

- We have presented two different discretization methods for a three-body system, THO and bin, based on expansion in HH.
- We have generalized the CDCC formalism for the application to four-body reactions.
- The formalism has been applied to the Borromean nucleus ^{6}He.
- We have seen as CDCC calculations with THO or bin as discretization methods is an efficient procedure for the study of four-body reactions.